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CARBON SYSTEMS

Electron States in Single-Layer Graphene Containing Short-Range Defects: The Potential Separable in the Momentum Representation

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Abstract—Electron states in single-layer graphene containing short-range defects are studied. A model potential concentrated on a circle and asymmetric with respect to the band number is treated. The study is carried out for the (2 + 1)-dimensional Dirac equation with a potential separable in the angular momentum representation. The characteristic equation derived in the study for bound and resonance states is compared with the equation derived previously for the same system by another method. The momentum representation used in this study provides a means for satisfactorily regularizing the Hadamard-incorrect problem with a singular potential. The method developed here is applied to regularization of the previously derived formulas for the scattering parameters and conductivity in graphene.

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1. INTRODUCTION

In the last few years, studies of the electron spectrum of graphene attracted considerable attention (see, e.g., [1] for a review). Points defects in graphene were considered in [2–4]. In [5, 6] we proposed a new model that takes into account the asymmetry of matrix elements of the defect potential with respect to the band number. This means that in the general case, the perturbation is described by a Hermitian matrix. We use a representation that provides the diagonal form of this matrix. Intervalley transitions are disregarded (see below). We treated the problem of electron states in single-layer graphene containing a point defect, the potential of which is concentrated on a circle, in [5, 6]. In that case we faced the following obstacle: the singular character of the model potential brought us to the issue of the correctness of the formulated problem, which is a specific property of the δ-shaped potential in the Dirac equation. Despite this obvious disadvantage, the δ-shaped potential is very extensively used in kinetic problems due to its formal simplicity and the minimum number of parameters. Therefore, the above-mentioned obstacle deserves a detailed analysis. The approach used in [5, 6] led us to a great number of solutions, among which there are probably phantom solutions. In this context, a demand has been created for some procedure of regularization that excludes such phantom solutions.

This study is concerned with the development of a procedure that makes it possible to obtain correct results for single-layer graphene in the case of use of a singular potential. The characteristic equation derived here for bound and resonance states is compared with the equation derived previously for the same problem by another method. The previously obtained formulas for the scattering amplitude and relaxation time are regularized. It is concluded that the method of regularization used in this study offers a correct formulation of the problem. Specifically, the spectrum depends mainly on variations in the conditions of matching.

The theory of transport processes involving resonance scattering and the theory of optical absorption in graphene demand for a nonperturbative approach to the analysis of electron states in the presence of defects. Because of this requirement, it is necessary to consider the models of defects that allow exact solutions. One such model for single-layer graphene is presented by the δ-shaped potential concentrated on a circle. The authors of [7] considered a two-band quasi-relativistic problem of bound and resonance states described by the Dirac equation in the case of a three-dimensional (3D) narrow- or zero-gap semiconductor containing such a defect. In that case the authors of [7] used the δ-function model. It is important that the δ-shaped potential has no singularity at $r = 0$ and is separable in the angular momentum representation. In this study we develop this approach applied to electron states in single-layer graphene. We take into account the possible asymmetry of the matrix elements of the defect potential with respect to band numbers (the asymmetry can appear due to local breakage of the symmetry of sublattices), which is equivalent to the introduction of a local perturbation

of the potential and mass. The purpose of this study is to regularize this Hadamard-incorrect problem. The procedure developed here is applied to regularization of the formulas for the conductivity of graphene.

2. FORMULATION OF THE PROBLEM IN THE MOMENTUM REPRESENTATION IN THE CASE OF A SEPARABLE POTENTIAL

The Dirac equation that describes electron states in graphene is

$$\left[-i\hbar v_F \sum_{\mu=1}^{2} \gamma_\mu \partial_\mu - \gamma_0 (m + \delta m) v_F^2\right] \psi = (E - V)\psi, \quad (1)$$

where v_F is the Fermi velocity of band electrons; γ_μ are the Dirac matrices $\gamma_0 = \sigma_3$, $\gamma_1 = \sigma_1$, and $\gamma_2 = i\sigma_2$; σ_i are the Pauli matrices; $2m v_F^2 = E_g$ is the band gap; and $\psi(\mathbf{r})$ is a two-component spinor,

$$\psi(\mathbf{r}) = \begin{pmatrix} f(\mathbf{r}) \\ g(\mathbf{r}) \end{pmatrix}. \quad (2)$$

The band gap can appear in the electron spectrum of graphene, e.g., due to some relative displacement of sublattices under the influence of the substrate [8]. The spinor structure takes into account the graphene structure consisting of two sublattices. In Eq. (1) $\delta m(\mathbf{r})$ and $V(\mathbf{r})$ are local perturbations of the mass (band gap) and potential. The local perturbation of the mass can be induced by a defect in the graphene film or at the substrate surface [8]. We here consider the δ-function model of the defect:

$$\delta m(\mathbf{r}) = -b\delta(r - r_0), \quad V(\mathbf{r}) = -a\delta(r - r_0). \quad (3)$$

Here, r and r_0 are the radial coordinate and the perturbation radius, respectively. Such short-range perturbation was used to solve the problem of bound and resonance states in 3D narrow-gap or zero-gap semiconductors. In some cases it is more convenient to introduce the potential asymmetric with respect to bands

$$\mathrm{diag}(V_1, V_2), \quad (4)$$

where

$$V_1 = V_1^0 \delta(r - r_0), \quad V_2 = V_2^0 \delta(r - r_0), \quad (5)$$

instead of the perturbations of the mass and potential.

The components of potential (4) are related to the parameters a and b by the relations

$$-V_1^0 = a + b, \quad -V_2^0 = a - b. \quad (6)$$

The perturbation in the form of the δ functions presents a simple model of a short-range defect. The finite radius r_0 makes it possible to regularize the formulated problem. The difficulty is that the δ-shaped potential makes the Dirac problem incorrect in Hadamard's sense [9]. In particular this means that the results may depend on the order of passages to the limit. The finiteness of the radius r_0 makes it possible also to exclude deep states that are nonphysical in the continuous approximation. The finiteness of the radius r_0 brings about the appearance of a form factor of the potential in the momentum representation such that it makes the scattering process with transfer of a large momentum (on the order of the distance between the points K and K' in the Brillouin zone) inefficient. Consequently, in this case, it is possible to disregard intervalley transitions [7].

Let us introduce the two-dimensional Fourier transformation of two-component wave function (2):

$$f(\mathbf{r}) = \int \frac{dp_x dp_y}{(2\pi)^2} e^{i\mathbf{kr}} f_\mathbf{p}, \quad g(\mathbf{r}) = \int \frac{d^2 p}{(2\pi)^2} e^{i\mathbf{kr}} f_\mathbf{p}. \quad (7)$$

Here the Fourier components $f_\mathbf{p}$ and $g_\mathbf{p}$ are determined by the inverse transform:

$$f_\mathbf{p} = \int dx dy e^{-i\mathbf{kr}} f(\mathbf{r}), \quad g_\mathbf{p} = \int dx dy e^{-i\mathbf{kr}} g(\mathbf{r}). \quad (8)$$

The inverse Fourier transform can be represented as a combination of the Hankel transform and the Fourier series in the angular variable [10]:

$$\begin{aligned} f_\mathrm{p} \equiv f(p, \theta) &= \sum_{n=-\infty}^{\infty} i^n e^{in\theta} f_n(p), \\ g_\mathrm{p} \equiv g(p, \theta) &= \sum_{n=-\infty}^{\infty} i^n e^{in\theta} g_n(p), \end{aligned} \quad (9)$$

$$\begin{aligned} f_n(p) &= \int_0^\infty dr r f_n(r) J_n(pr), \\ g_n(p) &= \int_0^\infty dr r g_n(r) J_n(pr). \end{aligned} \quad (10)$$

Here, $f_n(r)$ is the angular Fourier component,

$$\begin{aligned} f(r, \theta) &= \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} f_n(r) e^{in\theta}, \\ g(r, \theta) &= \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} g_n(r) e^{in\theta}. \end{aligned} \quad (11)$$

In passing to the Fourier transform of Dirac equation (1), we preliminarily consider two terms that require special attention. The gradient-involving terms are

$$-i(\hat{\sigma}_x \partial_x + \hat{\sigma}_y \partial_y)\psi(\mathbf{r}) = \begin{pmatrix} (\hat{p}_x + i\hat{p}_y) g(\mathbf{r}) \\ (\hat{p}_x - i\hat{p}_y) f(\mathbf{r}) \end{pmatrix}. \quad (12)$$

Consequently, in the momentum representation, the gradient-involving terms for the upper and lower spinor components take the form

$$\begin{pmatrix} (p_x + ip_y)g(\mathbf{p}) \\ (p_x - ip_y)f(\mathbf{p}) \end{pmatrix} = \begin{pmatrix} pe^{i\theta}g(p,\theta) \\ pe^{-i\theta}f(p,\theta) \end{pmatrix}. \quad (13)$$

Substituting (9) into (13), we obtain

$$\begin{pmatrix} +i\sum_{n=-\infty}^{\infty} i^n e^{in\theta} g_{n+1}(p) \\ -i\sum_{n=-\infty}^{\infty} i^n e^{in\theta} f_{n-1}(p) \end{pmatrix}. \quad (14)$$

The Fourier transform of the circularly symmetric potential, $V_i(\mathbf{p})$, can be expanded into series:

$$V_i(|\mathbf{p}-\mathbf{p}'|) = \pi\sum_{n=-\infty}^{\infty} e^{in\theta} V_i^n(p,p'), \quad (15)$$

$$V_i^n(p,p') = \int_0^{\infty} drrJ_n(pr)V_i(r)J_n(p'r). \quad (16)$$

Here, $J_n(pr)$ is the Bessel function and $i = 1, 2$. Thus, we obtain a set of integral equations:

$$(E-m)f_j(p) + ipg_j(p) - \int dp'p'f_j(p')V_1^j(p',p) = 0, \quad (17)$$

$$(E+m)g_j(p) - ipf_j(p) - \int dp'p'g_j(p')V_2^j(p',p) = 0. \quad (18)$$

We set $n = j - 1/2$, where j is the pseudospin quantum number; $j = \pm 1/2, \pm 3/2, \ldots$. In contrast to j in the relativistic theory, the quantum number j has nothing in common with the spin and describes the degeneracy at the Dirac point. The symmetry of Eqs. (17) and (18) is characterized as

$$f_j \leftrightarrow g_j, \quad E \to -E, \quad j - 1/2 \to j + 1/2, \quad a \to -a. \quad (19)$$

The symmetry of Eqs. (17) and (18) represents two symmetries inherent in graphene, specifically, the exact symmetry with respect to two sublattices and the approximated symmetry of bands (charge symmetry) that is valid in the limit of a small deviation of the momentum from the Dirac point.

Zero-radius potentials [12] and separable potentials [12] are widely used in nonrelativistic scattering theory. At the same time, the Dirac equation is extremely sensitive to potential singularities [13]. The singularity of the δ-shaped potential can be weakened by specifying the δ function on circle (5) [7]. Substituting (5) into (15), we obtain the potential separable in the angular momentum representation:

$$V_i^j(p,p') = v_i^j(p)v_i^j(p'), \quad i = 1, 2. \quad (20)$$

Here

$$v_1^j(p) = \sqrt{r_0 V_1^0} J_{j-1/2}(pr_0),$$
$$v_2^j(p) = \sqrt{r_0 V_2^0} J_{j+1/2}(pr_0). \quad (21)$$

The finiteness of the potential radius does not mean that the effective potential is concentrated within one unit cell. The matrix elements of the potential in the Kohn–Luttinger representation (this being the representation that corresponds to the standard Dirac representation) can be of considerably larger radius. In the model treated here, this means that the inequality $r_0 > a$ is valid. From (19) it is evident that, if this inequality is satisfied, the form factor of the potential is such that transitions between different Dirac points are improbable.

Equations (17) and (18) become degenerate and can be written as

$$(m-E)f_j(p) + pg_j(p) + v_1^j(p)\int_0^{\infty} dp'p'f_j(p')v_1^j(p') = 0, \quad (22)$$

$$(m-E)f_j(p) + pg_j(p) - v_2^j(p)\int_0^{\infty} dp'p'g_j(p')v_2^j(p') = 0. \quad (23)$$

3. REGULARIZATION OF THE CHARACTERISTIC EQUATION

We introduce the functions

$$F_j(E) = \int_0^{\infty} dp'pf_j(p)v_1^j(p),$$
$$G_j(E) = \int_0^{\infty} dp'pg_j(p)v_2^j(p), \quad (24)$$
$$R(p) = (p^2 + m^2 - E^2)^{-1}.$$

Then we obtain the set of algebraic equations

$$F_j = G_j\int_0^{\infty} dp'p^2R(p)v_1^j(p)v_2^j(p)$$
$$-(E+m)F_j\int_0^{\infty} dp'pR(p)[v_1^j(p)]^2, \quad (25)$$

$$G_j = F_j\int_0^{\infty} dp'p^2R(p)v_1^j(p)v_2^j(p)$$
$$+(E-m)G_j\int_0^{\infty} dpp^2R(p)[v_2^j(p)]^2. \quad (26)$$

The condition for solvability of this set of equations yields a characteristic equation, the roots of which define the bound and resonance states:

$$\left\{1 + (m+E)\int_0^\infty dpp R(p)[v_1^j(p)]^2\right\} \times \left\{1 + (m-E)\int_0^\infty dpp R(p)[v_2^j(p)]^2\right\} = \left[\int_0^\infty dpp^2 R(p) v_1^j(p) v_2^j(p)\right]^2. \quad (27)$$

Using the well-known formula [14]

$$\int_0^\infty dx \frac{x^{\mu-\nu+2n+1}}{x^2+z^2} J_\mu(bx) J_\nu(cx) = (-1)^n z^{\mu-\nu+2n} I_\nu(x) K_\mu(x), \quad (28)$$

we obtain

$$[1 + (m+E)V_1^0 I_{j-1/2}(\kappa r_0) K_{j-1/2}(\kappa r_0)] \times [1 + (m-E)V_2^0 I_{j+1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0)] = (m+E)(m-E) I_{j-1/2}^2(\kappa r_0) K_{j+1/2}^2(\kappa r_0), \quad (29)$$

where $\kappa^2 = (m^2 - E^2)$ and $I_n(x)$ and $K_n(x)$ are the modified Bessel functions. Using the identity [15]

$$I_\nu(x) K_{\nu+1}(x) + I_{\nu+1}(x) K_\nu(x) = 1/x, \quad (30)$$

we obtain the characteristic equation

$$\kappa[I_{j-1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0) + I_{j+1/2}(\kappa r_0) K_{j-1/2}(\kappa r_0)] + (m+E)V_1^0 I_{j-1/2}(\kappa r_0) K_{j-1/2}(\kappa r_0) + (m-E)V_2^0 I_{j+1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0) = 0. \quad (31)$$

Using relation (6), we obtain the equation

$$\kappa[I_{j-1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0) + K_{j-1/2}(\kappa r_0) I_{j+1/2}(\kappa r_0)] = [(m-E)(a-b) I_{j+1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0) + (a+b)(m+E) I_{j-1/2}(\kappa r_0) K_{j-1/2}(\kappa r_0)]. \quad (32)$$

Let us compare Eq. (32) with the equation

$$p[J_{j-1/2}(pr_0) H_{j+1/2}^{(1)}(pr_0) - J_{j+1/2}(pr_0) H_{j-1/2}^{(1)}(pr_0)] = T(a,b)\left[\sqrt{\frac{E-m}{E+m}}(a-b) J_{j+1/2}(pr_0) H_{j+1/2}^{(1)}(pr_0) + \sqrt{\frac{E+m}{E-m}}(a+b) J_{j-1/2}(pr_0) H_{j-1/2}^{(1)}(pr_0)\right], \quad (33)$$

obtained by us previously by another method [6]. Here

$$T(a,b) = \frac{\tan(\sqrt{a^2-b^2})}{\sqrt{a^2-b^2}}, \quad (34)$$

$H_n^{(1)}(z)$ is the Hankel function, and $p = \sqrt{E^2 - m^2}$. Accomplishing the analytic continuation from the case of band states with $E^2 > m^2$ to the opposite case of bound states with $E^2 < m^2$, we obtain the equation

$$\kappa[I_{j-1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0) + K_{j-1/2}(\kappa r_0) I_{j+1/2}(\kappa r_0)] = T(a,b)[m - E(a-b) I_{j+1/2}(\kappa r_0) K_{j+1/2}(\kappa r_0) + (a+b)(m+E) I_{j-1/2}(\kappa r_0) K_{j-1/2}(\kappa r_0)]. \quad (35)$$

We can see that formulas (32) and (35) differ only by factor (34) entering into the equation derived in [6]. Equations (32) and (35) are practically identical in the limit of

$$a^2 - b^2 \to 0, \quad T(a,b) \to 1. \quad (36)$$

This limit can be attained on the line $a^2 - b^2 = 0$ or with small a and b. The cause of the difference between Eqs. (32) and (35) can be understood from the consideration presented below. The δ-shaped potential involved in the Dirac equation gives rise to a boundary problem that is incorrect in Hadamard's sense [9]. This problem needs to be regularized. In doing so we use different approaches in this study and in the study reported in [6]. In [6] the singularity of the perturbation was retained and the regularization was accomplished with the use of a special procedure of matching the solutions, whereas in this study, the regularization is attained due to the smoothing property of the Fourier transform. Specifically, the potential takes a separable form with the nonlocal kernel $v_i^l(p) = \sqrt{r_0 V_i^0} J_{l+1/2}(pr_0)$ (see formula (21)). It is this nonlocality that plays the major role in regularization. Equations (32) and (35) have asymptotically coinciding solutions in the limit described by (36). However, the sets of solutions of Eqs. (32) and (35) are different outside the region of the parameters a and b defined by (36). Moreover, Eq. (35) has a much richer set of solutions than Eq. (32) [6] because of the periodicity of the tangent in (34). We come to the conclusion that the richer set of solutions of Eq. (35) is an artifact of the model of point interaction. Finally, the model of a ring-shaped potential well treated in [16] does not reproduce this rich set of solutions even in the limit of the zero-width ring and an infinitely deep well. Consequently, the equation derived in this study can be considered as the solution of a correctly regularized boundary problem.

4. THE REGULARIZED SCATTERING MATRIX AND RELAXATION TIME

The approach developed above can be applied to regularization of the problem of scattering at a point defect in graphene. Our purpose here is to correct the previously obtained formulas [6] for the parameters of scattering in this case with the regularization accomplished above. We define the scattering matrix element as the ratio between the amplitudes of the diverging and converging cylindrical waves corresponding to the solutions of integral Eqs. (17) and (18). Then we obtain

$$S_j(E) = -\frac{\mathcal{F}_j^{(2)}}{\mathcal{F}_j^{(1)}}. \tag{37}$$

Here the functions $\mathcal{F}_j^{(a)}$ are defined by the formula

$$\begin{aligned}\mathcal{F}_j^{(a)} &= -\sqrt{\frac{E-m}{E+m}}(a-b)J_{j+1/2}(pr_0)H^{(\alpha)}_{j+1/2}(pr_0)\\ &\quad -\sqrt{\frac{E+m}{E-m}}(a+b)J_{j-1/2}(pr_0)H^{(\alpha)}_{j-1/2}(pr_0)\\ &\quad + p[J_{j-1/2}(pr_0)H^{(\alpha)}_{j+1/2}(pr_0) - J_{j+1/2}(pr_0)H^{(\alpha)}_{j-1/2}(pr_0)].\end{aligned} \tag{38}$$

The above functions differ from the previously derived functions [6]

$$\begin{aligned}\mathcal{F}_j^{(a)} &= -T(a,b)\Bigg[\sqrt{\frac{E-m}{E+m}}(a-b)J_{j+1/2}(pr_0)H^{(\alpha)}_{j+1/2}(pr_0)\\ &\quad +\sqrt{\frac{E+m}{E-m}}(a+b)J_{j-1/2}(pr_0)H^{(\alpha)}_{j-1/2}(pr_0)\Bigg]\\ &\quad + p[J_{j-1/2}(pr_0)H^{(\alpha)}_{j+1/2}(pr_0) - J_{j+1/2}(pr_0)H^{(\alpha)}_{j-1/2}(pr_0)]\end{aligned} \tag{39}$$

by the absence of the factor $T(a, b)$ (formula (34)) in the regularized expression. Using the well-known formulas for the Hankel functions $H_n^{(1)}(z) = J_n + iN_n$ and $H_n^{(2)}(z) = J_n - iN_n$, we can write the regularized S matrix as

$$S_j(E) = -\frac{A_j(E) - iB_j(E)}{A_j(E) + iB_j(E)} = \frac{B_j(E) + iA_j(E)}{B_j(E) - iA_j(E)}; \tag{40}$$

hence, this matrix can be represented in the standard form [17]

$$S_j(E) = \exp[i2\delta_j(E)]. \tag{41}$$

Here the scattering phase is defined by the expression

$$\delta_j(E) = \arctan\frac{A_j(E)}{B_j(E)}. \tag{42}$$

Formulas (40) and (41) show that the regularized S matrix is unitary. The functions $A_j(E)$ and $B_j(E)$ are written as

$$\begin{aligned}A_j(E) &= (a+b)\sqrt{\frac{E+m}{E-m}}J^2_{j-1/2}(pr_0)\\ &\quad + (a-b)\sqrt{\frac{E-m}{E+m}}J^2_{j+1/2}(pr_0),\end{aligned} \tag{43}$$

$$\begin{aligned}B_j(E) &= (a+b)\sqrt{\frac{E+m}{E-m}}J_{j-1/2}(pr_0)N_{j-1/2}(pr_0)\\ &\quad + (a-b)\sqrt{\frac{E-m}{E+m}}J_{j+1/2}(pr_0)N_{j+1/2}(pr_0)\\ &\quad + p[J_{j-1/2}(pr_0)N_{j+1/2}(pr_0) - J_{j+1/2}(pr_0)N_{j-1/2}(pr_0)].\end{aligned} \tag{44}$$

Using the expansion of the Bessel functions [18]

$$J_n(x) \sim (1/n!)(x/2)^n, \tag{45}$$

$$N_n(x) \sim \begin{cases}-[\Gamma(n)/\pi](2/x)^n & \text{at} \quad n > 0,\\ (2\pi)\ln(\gamma_E x/2) & \text{at} \quad n = 0,\end{cases} \tag{46}$$

we come to the conclusion that, for low-energy scattering at a short-range potential ($pr_0 \ll 1$), the phase $\delta_j(E)$ is small as $(pr_0)^{|j|+1/2}$, excluding the case of $j = \pm 1/2$. Here $\ln\gamma_E$ is the Euler constant. If the radius r_0 and the energy E are small, we can disregard all higher partial waves, taking into account only the phase δ_j for $j = \pm 1/2$:

$$\tan\delta_{1/2}(E) = \sqrt{\frac{E+m}{E-m}}(a+b)\pi\left(\frac{pr_0}{2}\right), \quad pr_0 \to 0, \tag{47}$$

$$\tan\delta_{-1/2}(E) \approx \sqrt{\frac{E-m}{E+m}}(a-b)\pi\left(\frac{pr_0}{2}\right), \quad pr_0 \to 0. \tag{48}$$

Thus, the phase is proportional to pr_0 in the long-wavelength limit, in accordance with the general principles of quantum mechanics [3, 17]. The scattering amplitude $f(\theta)$ and the transport scattering cross section Σ_{tr} can be expressed in terms of $S_j(E)$ as [3]

$$\begin{aligned}f(\theta) &= \frac{1}{i\sqrt{2\pi p}}\\ &\quad \times \sum_{j = \pm 1/2, \pm 3/2, \ldots}[S_j(E) - 1]\exp[i(j-1/2)\theta],\end{aligned} \tag{49}$$

$$\Sigma_{tr} = 2/p \sum_{j = \pm 1/2, \pm 3/2, \ldots} \sin^2(\delta_{j+1} - \delta_j). \tag{50}$$

Near the resonance, the expression for the phase takes the Breit–Wigner form [17]: $\delta_j \approx \delta_j^{(0)} + \arctan\frac{\Gamma_j}{2(E_j^{(0)} - E)}$. Here $E_j^{(0)}$ and Γ_j are the position and width of the resonance level, respectively, and $\delta_j^{(0)}$ is the slowly varying phase of the potential (background) scattering.

The above formulas can be used to calculate the Boltzmann conductivity (see [19])

$$\sigma = \left(\frac{e^2}{2\pi\hbar}\right)\frac{2E_F}{\hbar}\tau_{tr}, \quad (51)$$

where the transport relaxation time τ_{tr} is determined by the formula

$$1/\tau_{tr} = N_i v_F \Sigma_{tr}. \quad (52)$$

Here, N_i is the concentration of impurities per unit area and $E_F = v_F\kappa_F$. The above expressions convert the dependence of the parameters of scattering on the Fermi energy and on the parameters of scatterers a and b to the corresponding dependence of the Boltzmann conductivity. Thus, the approach developed here provides a means for studying the influence of the specific features of the parameters of scattering of electrons in graphene on the behavior of the electrical conductivity.

5. CONCLUSIONS

It is shown that the method of Fourier transform of the Dirac equation with the δ-shaped potential asymmetric with respect to bands yields a regularized problem, for which there are no extra phantom solutions that are an artifact of the singular potential. This approach regularizes the Hadamard-incorrect problem. The method developed in this study is applied to regularization of the previously derived formulas for the parameters of scattering and the conductivity.

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