Федеральное государственное бюджетное образовательное учреждение высшего профессионального образования
САНКТ-ПЕТЕРБУРГСКИЙ ГОСУДАРСТВЕННЫЙ ПОЛИТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ

INSTITUTE OF APPLIED MATHEMATICS AND MECHANICS
DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS

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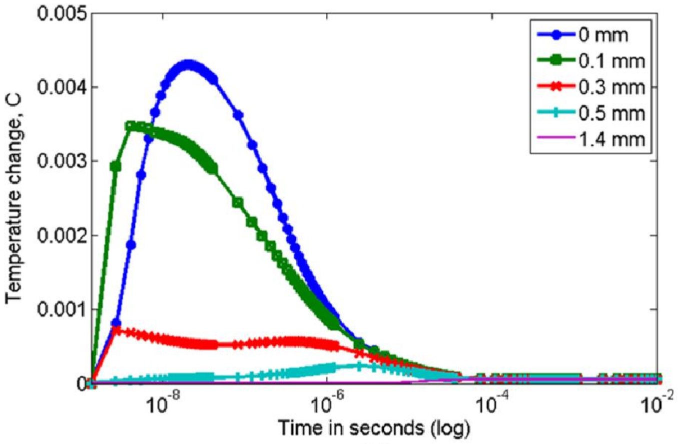
LECTURE 9

Models of the laser radiation impact on the skin

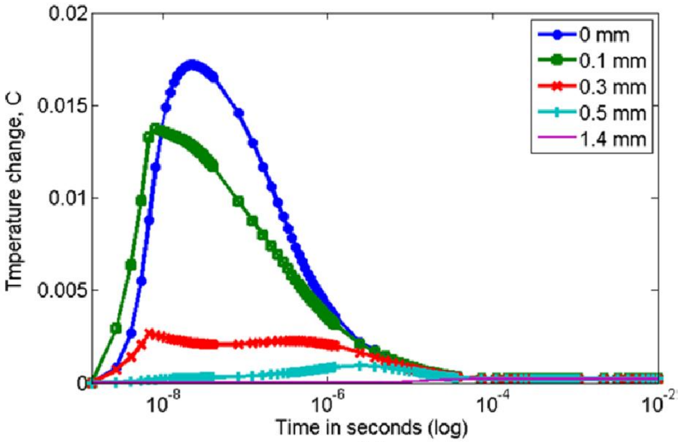
Lecture slides
for Bachelors of Technical Sciences

Санкт-Петербургский государственный политехнический университет
2012

Laser impact studies

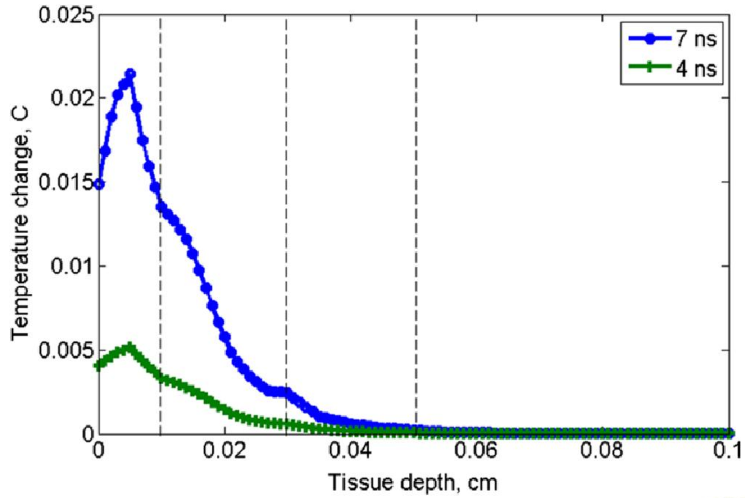


(a)



(b)

The analysis of the results showed that heat is not localized on the surface, but is collected inside the tissue in lower skin layers. The simulation was made with the pulsed UV laser beam (used as excitation source in laser-induced fluorescence) and the cw visible laser (used in photodynamic therapy treatments), in order to study the possible thermal effects.



Dynamic model of thermal reaction of biological tissues to laser-induced fluorescence and photodynamic therapy, Alexey Yu. Seteikin et. al., 2013

The thermally insulated layer

- Differential equation for the heat flux can be written as follows:

$$\dot{\eta} + \tau \ddot{\eta} - \eta'' = \gamma H(t-0) \mathrm{e}^{-\gamma \varsigma}$$

- The notations, boundary, and initial conditions are the same. Following the procedures, we obtain an expression for the temperature:

$$\vartheta = \mathrm{e}^{-\varsigma\gamma} t H(t) - 2\mu\gamma \mathrm{e}^{-\mu\gamma} \sum_{n=1}^{\infty} \left(\frac{\left((-1)^n - \mathrm{e}^{\mu\gamma}\right) \cos\left[\frac{n\pi\varsigma}{\mu}\right]}{n^2\pi^2 \left(n^2\pi^2 + \mu^2\gamma^2\right)} \left(\mu^2 - n^2\pi^2 t \right.\right.$$

$$\left.\left. - \mathrm{e}^{-\frac{t}{2\tau}} \mu \left(\mu \cos\left[\frac{t S_n}{2\tau\mu}\right] + \frac{\mu^2 - 2n^2\pi^2\tau}{S_n} \sin\left[\frac{t S_n}{2\tau\mu}\right] \right) \right) \right) H(t)$$

The thermally insulated layer

- A solution of this problem in the classic formulation is:

$$\vartheta = \frac{\left(1-e^{-\mu\gamma}\right)t}{\mu\gamma}H(t)$$
$$+2\mu^3\gamma\sum_{n=1}^{\infty}\frac{\left(e^{\mu\gamma}-(-1)^n\right)\cos\left[\frac{n\pi\varsigma}{\mu}\right]e^{-\mu\gamma}\left(1-\exp\left(-\frac{n^2\pi^2 t}{\mu^2}\right)\right)}{n^2\pi^2\left(n^2\pi^2+\mu^2\gamma^2\right)}H(t)$$

- In the limit case of the wave equation, an expression for ϑ is written as follows:

$$\vartheta = e^{-\varsigma\gamma}tH(t)-2\mu\gamma e^{-\mu\gamma}$$
$$\times\sum_{n=1}^{\infty}\frac{\left((-1)^n-e^{\mu\gamma}\right)\cos\left[\frac{n\pi\varsigma}{\mu}\right]\left(\mu\sqrt{\tau}\sin\left[\frac{n\pi t}{\mu\sqrt{\tau}}\right]-n\pi t\right)}{n\pi\left(n^2\pi^2+\mu^2\gamma^2\right)}H(t)$$

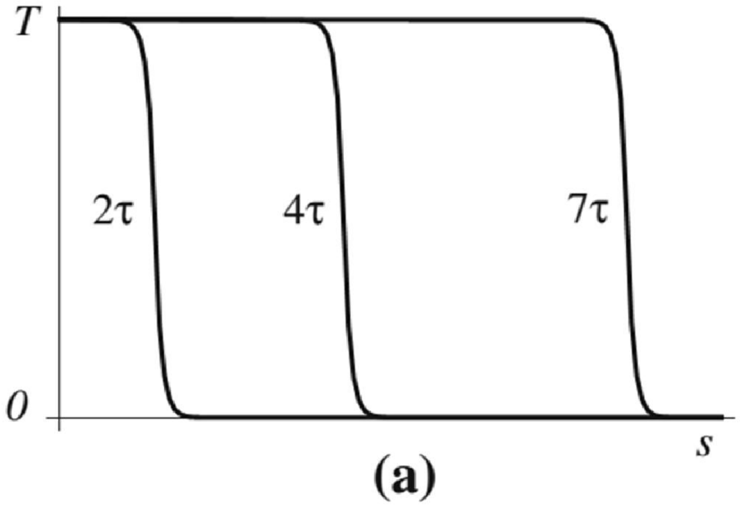
The wave front

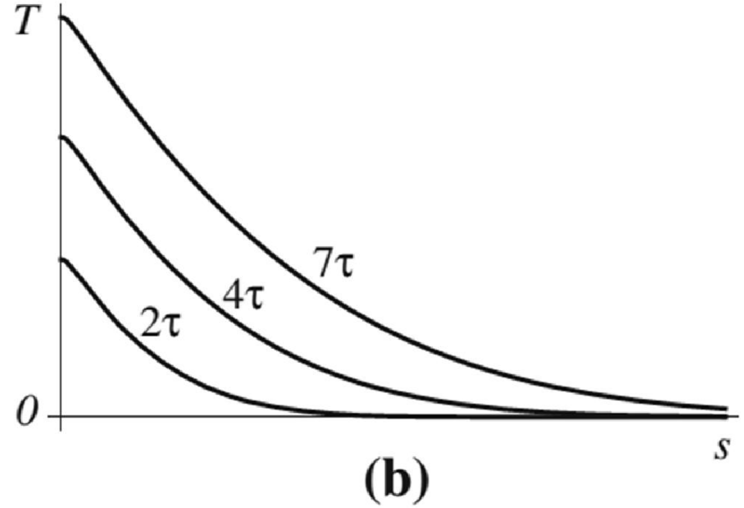
- The temperature value on the wave front can be obtained as a special case of the solution for the half-space:

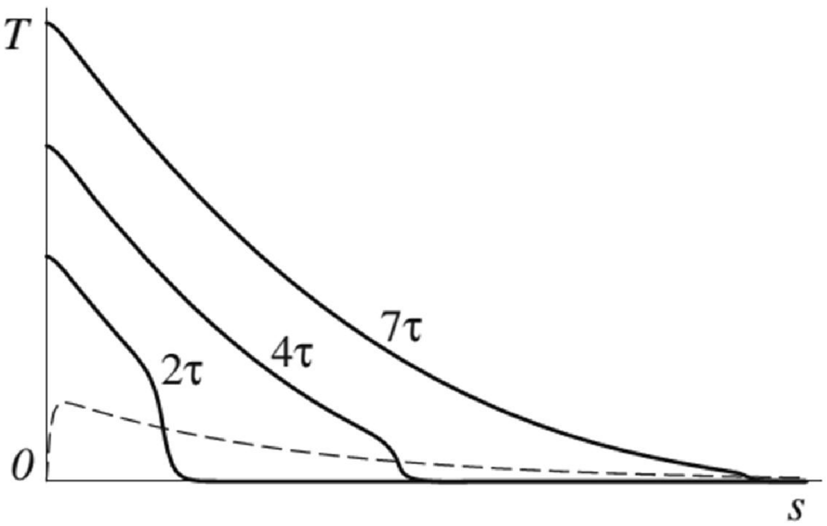
$$\vartheta_3(\varsigma) = \frac{1}{2\gamma^2 r} e^{-\frac{(r+1)x}{2\sqrt{\tau}} - \gamma x} \left(\left(2\gamma^2\tau + r + 1\right) e^{\frac{rx}{\sqrt{\tau}}} - 2\gamma^2\tau - 2re^{\frac{(r+1)x}{2\sqrt{\tau}}} + r - 1 \right)$$

- The solution graphics of the hyperbolic heat conduction equation monotonously decrease at all values of time: they have no salient points or jumps, typical for the previously concerned problems.

Solution curves







Analysis

- If the attenuation of the laser radiation in the layer is significant, then the solution profiles are similar to the solution profiles at short times.
- Hyperbolic curves become similar to the curves for the classic heat conduction problem with increasing of time or with decreasing of the attenuation coefficient γ in the medium.
- Furthermore, the solution becomes similar to the solution of the boundary-value problem with increasing of the attenuation coefficient.
- T values quicker reach zero after its “wave front.” Classic solution curves are similar.

The boundaries are kept at a constant temperature

- Differential equation for the temperature with heat source is written as follows:

$$\dot{\vartheta} + \tau\ddot{\vartheta} - \vartheta'' = (H(t-0) + \tau\delta(t-0))\mathrm{e}^{-\gamma\varsigma}$$

- Following the procedure used before, we obtain an expression for the temperature:

$$\vartheta = 2\mu^2\mathrm{e}^{-\mu\gamma}\sum_{n=1}^{\infty}\frac{(\mathrm{e}^{\mu\gamma} - (-1)^n)\sin\left[\frac{n\pi\varsigma}{\mu}\right]}{n\pi\left(n^2\pi^2 + \mu^2\gamma^2\right)}$$
$$\times\left(1 - \mathrm{e}^{-\frac{t}{2\tau}}\left(\frac{\mu^2 - 2n^2\pi^2\tau}{\mu S_n}\sin\left[\frac{tS_n}{2\tau\mu}\right] + \cos\left[\frac{tS_n}{2\tau\mu}\right]\right)\right)H(t)$$

The limiting cases

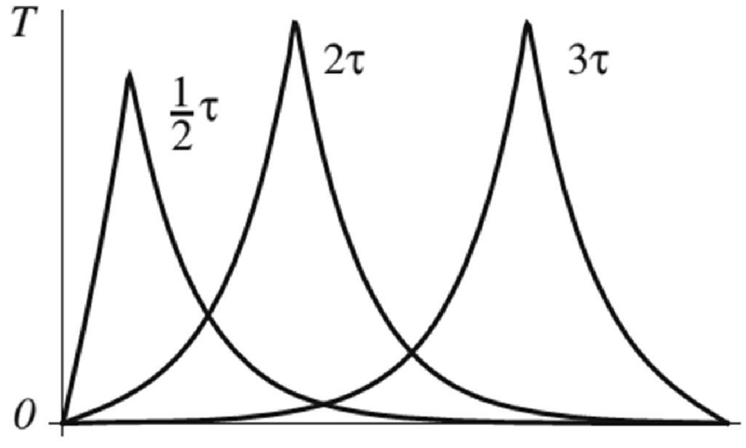
- A solution of the problem in the classic formulation can be written as follows:

$$\vartheta = \sum_{n=1}^{\infty} \frac{2\left(-(-1)^n + \mathrm{e}^{\mu\gamma}\right)\mu^2 + \mathrm{e}^{-\mu\gamma}\left(1 - \exp\left(-\frac{n^2\pi^2 t}{\mu^2}\right)\right)\sin\left[\frac{n\pi\varsigma}{\mu}\right]}{n\pi\left(n^2\pi^2 + \mu^2\gamma^2\right)} H(t)$$

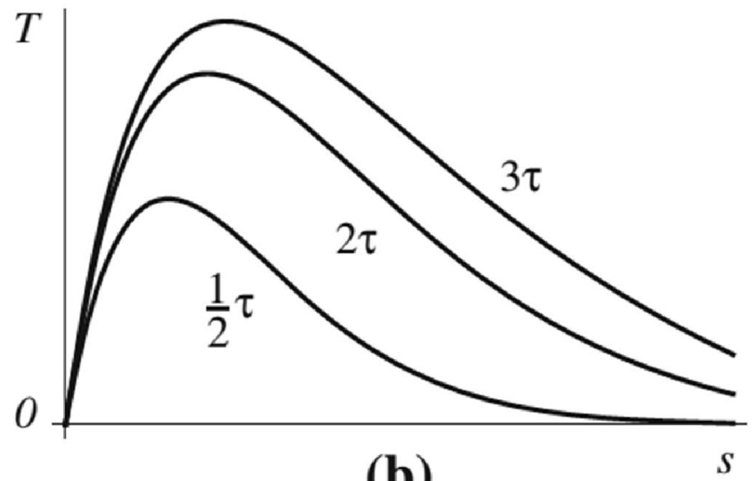
- In the limit case of the wave equation, an expression for ϑ is written as follows:

$$\vartheta = 2\mu\sqrt{\tau}\mathrm{e}^{-\mu\gamma} \sum_{n=1}^{\infty} \frac{\left(-(-1)^n + \mathrm{e}^{\mu\gamma}\right)\sin\left[\frac{n\pi\varsigma}{\mu}\right]\sin\left[\frac{n\pi t}{\mu\sqrt{\tau}}\right]}{n^2\pi^2 + \mu^2\gamma^2} H(t)$$

Solution curves



(a)



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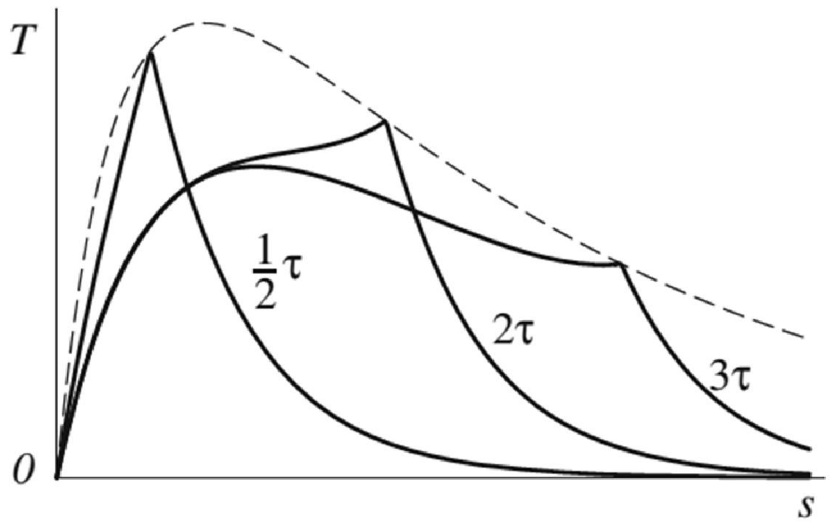

Analisys

- Unlike the temperature graphics for the layer under the influence of the short laser impulse, the graphics do not have the region of negative temperatures in the vicinity of the irradiated surface of the layer.
- For short times, the solution profile of the hyperbolic equation is similar to the solution profile of the wave equation: the temperature reaches its maximum at the salient point and then quickly decays.
- With increasing of time, a second extremum point which is similar to the extremum point of the classic curves appears and the salient point ceases to be a global maximum.