Numerical Simulation of Circumsolar Ring Evolution

A. S. Murachev, D. V. Tsvetkov, E. M. Galimov and A. M. Krivtsov

Abstract The results of the computer simulation for the circumsolar gas-dust cloud evolution are presented. The particle dynamics method is used. We show that gas-dust clusters can be formed in ring-shaped structures of protoplanetary disks. It is demonstrated that the clusters are formed as a result of the counteracting of the self-gravitational force of the ring and the gravity of the Sun. This process has a probabilistic nature. The range of the system parameters providing the clusters formation is obtained. Different scenarios of the ring evolution are observed and analyzed. The considered gas-dust clusters can be precursors for the further formation of the planet-satellite systems.

1 Introduction

Gas and dust play an important role in a planet formation [5, 18]. According to existing astrophysical theories, the process of star formation leads to the appearance of an accretion gas-dust disks around young stars [1]. Under certain conditions, planetary

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A. S. Murachev (✉) · D. V. Tsvetkov · A. M. Krivtsov
Peter the Great Saint Petersburg Polytechnic University,
Politekhnicheskaya Street 29, St. Petersburg, Russia
e-mail: andrey.murachev@gmail.com

D. V. Tsvetkov
e-mail: dvtsvetkov@ya.ru

A. M. Krivtsov
e-mail: akrivtsov@bk.ru

E. M. Galimov
Vernadsky Institute of Geochemistry and Analytical Chemistry of Russian
Academy of Sciences, Kosygina Street 19, St. Petersburg, Russia
e-mail: galimov@geokhi.ru

A. M. Krivtsov
Institute for Problems in Mechanical Engineering, Russian Academy of Sciences,
V.O., Bolshoj pr., 61, St. Petersburg, Russia
e-mail: akrivtsov@bk.ru

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systems can be formed from such accretion disks [16]. Recently, the high resolution submillimeter ALMA telescope made it possible to obtain magnificent images of protoplanetary systems from other stars [2, 4]. The observations show bright dust rings in the disk, separated by dark gaps. The origin of these structures is not entirely clear and is still a matter of debate, but it is likely that such structures can be associated with the planets formation. In particular, ring-shaped structures can concentrate and also prevent solid particles accretion onto star [19, 21].

Thus, the probable scenario of the circumstellar disk evolution is formation of gas-dust rings, which is confirmed by the mentioned above observations and the computer simulations [6, 15, 20]. These rings can be treated as protoplanetary rings since the planet formation process is likely to take place in these high density areas. The asteroid ring in the Solar system also can be considered as a result of a protoplanetary ring evolution were the planet formation process for some reasons was not completed (probably due to the Jupiter influence).

In the present paper we present a computer model of the protoplanetary ring clustering. The paper is based on the hypothesis of academician E.M. Galimov of planets formation [9–11]. We assume that the solid phase early occurrence and growth is not the only way of the protoplanetary nebula evolution. The appearance of gravitational instabilities in protoplanetary disks along with particle growth leads to the formation of separate gas-dust clouds. These clouds collide with each other, grow in size, and after they reach an appropriate size and some additional conditions are fulfilled then planet-satellite systems can be formed as a result of their rotational collapse [11].

We use the particle dynamics method [13] to explore the protoplanetary disk evolution. The particles are represented as a point masses that interact with each other and with the central star. The method allows to find the numerical solution of the N-body problem as a function of time. One of the important peculiarities of this work in comparison with other works studying the cluster formation using the particles dynamics method [7, 8, 12, 15, 17] is the nature of the interparticle force. The considered force contains three main terms: the gravitational attraction, the soft gas-aided repulsion, and the dissipative term. We optimize the calculations with the modified Barnes-Hut algorithm [14] which in contrast to the classical algorithm [3] efficiently works for both short-range and long-range forces.

Each cluster can be considered as a separated gas-dust cloud in the protoplanetary disk. The previous computer simulations [11, 14] showed that under certain conditions, a planet with its satellite (e.g. the Earth-Moon system) can be formed as a result of a rotational collapse of such gas-dust cloud. In particulary, our results can be used as initial condition for the study of the Earth-Moon formation hypothesis [10, 11].

2 The Model

The protoplanetary ring is modeled as a set of particles interacting via prescribed forces. Initial particle co-ordinates have a uniform random distribution in an oblate spheroid with semi-axes $R_{\max}$ and R_s, where $R_s/R_{\max} \ll 1$ without central cylindrical

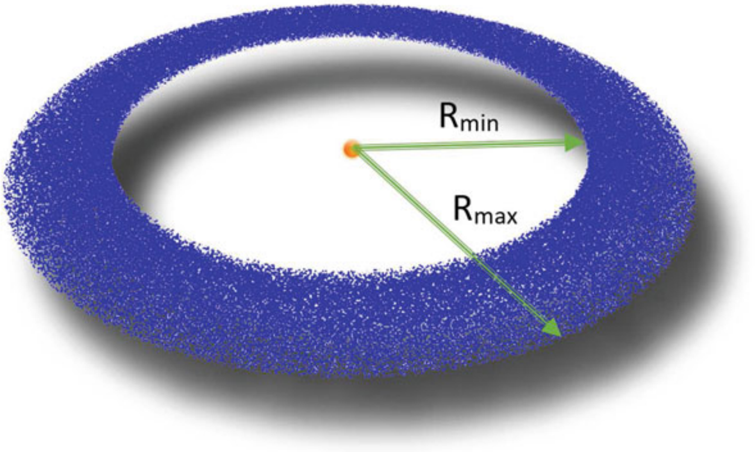

Fig. 1 The initial configuration of a protoplanetary disk

part with radius $R_{\min}$ (see Fig. 1). Further for simplicity directions in the ring plane will be called horizontal directions, the direction orthogonal to the ring plane—the vertical direction. The initial velocity of each particle is a sum of regular and random terms:

$$\boldsymbol{v} \overset{\text{def}}{=} \dot{\mathbf{r}} = \boldsymbol{v}_{\mathbf{reg}} + \boldsymbol{v}_{\mathbf{rand}}. \tag{1}$$

The regular component is defined as $\boldsymbol{v}_{\text{reg}} = \boldsymbol{\omega}_0 \times \boldsymbol{r}$, where $\boldsymbol{r}$ is the particle radius-vector and $\boldsymbol{\omega}_0$ is a vertical vector of the initial angular velocity of the particle. The absolute value of this vector is

$$\omega_0 = |\boldsymbol{\omega}_0| = \sqrt{\frac{\gamma M_{\text{sun}}}{r^3}}, \qquad r = |\boldsymbol{r}|, \tag{2}$$

where m is the particle mass, M_{sun} is the mass of the central star (further, for simplicity, the Sun), and γ is the gravitational constant. The random velocity components $\boldsymbol{v}_{\text{rand}}$ are randomly distributed in a spheroid with semi-axis $0.7v_{\text{reg}}$ and $0.055v_{\text{reg}}$ (horizontal and vertical, respectively), where $v_{\text{reg}} = |\boldsymbol{v}_{\text{reg}}|$.

The motion of the particles is described by the equations of Newtonian dynamics:

$$m\ddot{\boldsymbol{r}}_k = \sum_{n=1}^{N} \frac{1}{r_{kn}} f(r_{kn}, \dot{r}_{kn}) \boldsymbol{r}_{kn} + \boldsymbol{F}_k,$$

$$\boldsymbol{r}_{kn} = \boldsymbol{r}_k - \boldsymbol{r}_n, \qquad r_{kn} = |\boldsymbol{r}_{kn}|; \qquad k = 1, 2, \ldots, N, \tag{3}$$

where N is the number of particles, $\boldsymbol{r}_k$ is the radius-vector of the k^{th} particle, $f(r, \dot{r})$ is the interparticle interaction force and

$$\boldsymbol{F}_k = \gamma \frac{M_{\text{sun}} m}{r_k^3} \boldsymbol{r}_k \tag{4}$$

is the gravitational force between the Sun and k^{th} particle, and M_{sun} is the Sun mass. The main part of the interparticle force f is the gravitational attraction that is a long-range force. In addition, we assumed that each particle in the heated state emits gaseous substances. The evaporating gas forms gas shells around the particles. This leads to the mutual repulsion of the particles. Energy loss in the interaction of the gas shells is taken into account by introducing an additional dissipative force. Thus, the interaction force consists of three components: the gravitational attraction, the gas repulsions, and the dissipation [11]:

$$f(r, \dot{r}) = \frac{A_1}{r^2} + \frac{A_2}{r^p} + \frac{A_3}{r^q} \dot{r}. \tag{5}$$

The first term in Eq. (5) corresponds to the gravitational force, where $A_1 = \gamma m^2$. The second term is the short-range repulsive force, where $A_2 = -A_1 a^{p-2}$, a is the particle diameter (the equilibrium distance between two particles), and $p = 7$. The third term in Eq. (5) describes non-conservative interaction between particles, where $A_3 = \beta A_2$, $\beta = \sqrt{11a^3/(25\gamma m)}$, and $q = p + 1$. The value of β is chosen to have a moderate dissipation in the system, the value of q is chosen to keep proportional the second and the third terms in Eq. (5) when the interparticle distance is changed. Then Eq. (5) can be rewritten in the form [11]

$$f(r, \dot{r}) = \gamma \frac{m^2}{a^2} \left[\left(\frac{a}{r} \right)^p \left(1 - \beta \frac{\dot{r}}{r} \right) - \left(\frac{a}{r} \right)^2 \right]. \tag{6}$$

The evolution of the protoplanetary ring depends on many parameters. The main goal of the study is to define the values of parameters that govern clustering in the protoplanetary system. The following main dimensionless parameters can be outlined: the dimensionless disk internal radius $R_{\min}/a$, the shape ratio $W_r = R_{\max}/R_{\min}$, the mass ratio $M_s = M_{\text{sun}}/M$, and the dimensionless particle concentration n/n_0, where n is the average number of particle per unit volume, $n_0 = \frac{\pi}{3\sqrt{2}}$ is the particle close packing concentration. The oblateness $R_s/R_{\max} = 0.04$ for all the systems under study. Three values of the mass ratio $M_s = 5,\ 10,\ 20$ will be considered. The number of particles is set indirectly as a function of the dimensionless particle concentration and the dimensionless disk volume (volume divided by a^3). The average number of particles in our calculations is about 700 000.

We carried out about one and a half thousand calculations of the protoplanetary rings evolution. Each calculation takes time from 3 to 10 revolutions of a particle having an initial angular velocity ω_0.

3 The Numerical Method

The particle dynamics method allows to calculate trajectories of all the particles in the system. Therefore, it is able to describe the whole system evolution. However, taking into account interactions between all particles leads to the complexity $O(N^2)$ at each step of integration. For the modeling of the planetary system formation hundreds of thousands or even millions of particles are needed. Computer resources that are needed to model such systems in the case of the complexity $O(N^2)$ are unacceptable. The traditionally way out of this situation is the use of approximate calculations of interaction forces. The Barnes-Hut algorithm [3] is one of such methods. This is a hierarchical method that is based on combining close particles into groups and calculating the total potential approximation of the group. However, the classical Barnes-Hut method is suitable only for certain stages of the system evolution. In such stages, the particles should be distributed fairly evenly in space. When clusters are formed, where the concentrations of particles in order exceed the concentration in the surrounding space, the rate of calculation drops drastically.

In this paper we use the modified Barnes-Hut method [14] to solve this problem. The modified Barnes-Hut method accelerates the calculations for the case of a essentially inhomogeneous spatial particles distribution. The modified algorithm allows to increase substantially the calculation speed without loss of accuracy. The modified method was applied to calculate the evolution of the gas-dust cloud for studying Moon and Earth system formation [11].

4 The Simulation Results

4.1 The Protoplanetary Rings Evolution

Let us consider the evolution of one of the systems where the clustering occurs. Figure 2 shows radial distribution of the average particles concentration in sequential moments of time. The average particle concentration at a given radial distance is

$$n = \frac{N_r}{S_r a}, \tag{7}$$

where N_r is the number of particles at the distance $r+\Delta r$ from the system center, Δr is numerical step of the radius and $S_r = \pi(r+\Delta r)^2 - \pi r^2$ is the area of the median plane of the protoplanetary ring. Figure 2 shows 15 graphs corresponding to 15 consecutive moments of time. It is clearly seen that the peaks of concentration appear in the late stages of the protoplanetary ring evolution, where each peak corresponds to a cluster.

The computations show that if the clusters were not formed during several first orbital periods then they are never formed. This conclusion is confirmed by the nature

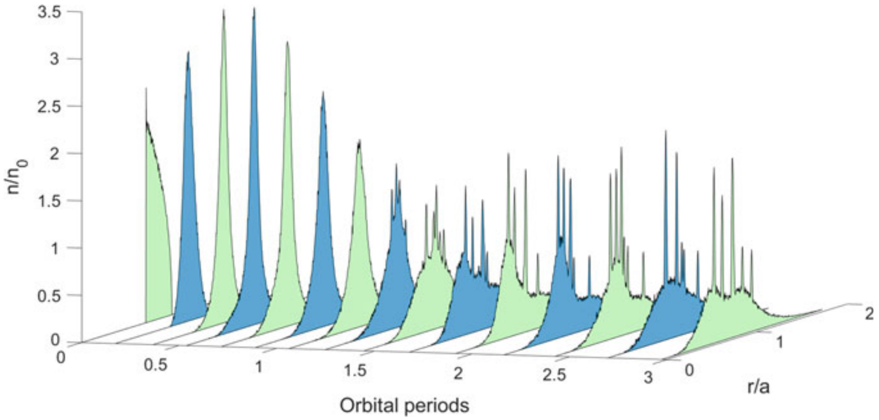

Fig. 2 Time evolution of the radial density in the ring. $R_{\min}/a = 200, W_r = 1.4, M_s = 10, n/n_0 = 0.1$

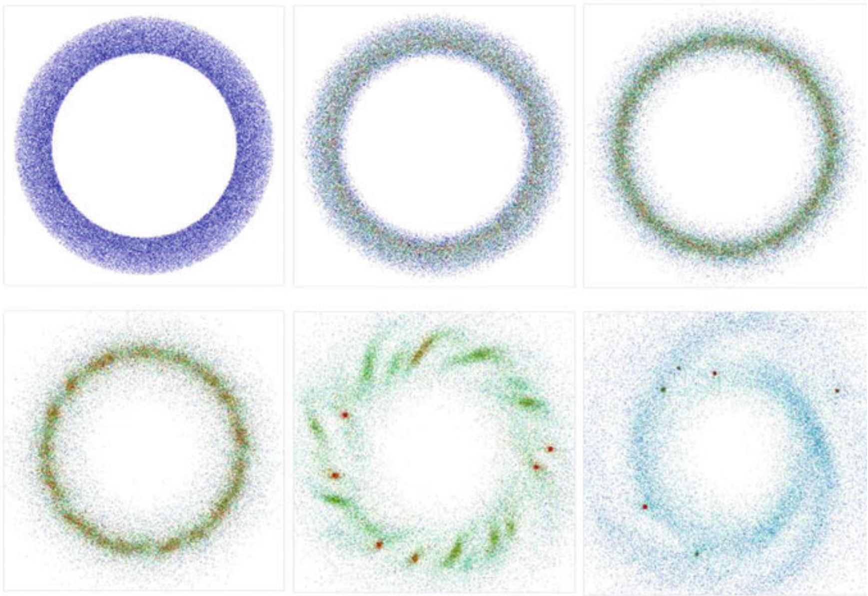

Fig. 3 The typical evolution of a protoplanetary ring. $R_{\min}/a = 200, W_r = 1.4, M_s = 10, n/n_0 = 0.1$

of the cluster formation in the ring. There are three stages in the evolution of the protoplanetary ring, which we can distinguish (see Fig. 3):

Stage 1. The stage of initial particles uniform distribution in the protoplanetary ring.
Stage 2. The stage of counteracting. Two processes occur simultaneously. The whole subsequent evolution of the system is determined by a process which is

more dominant. The first process is attraction of the particles to the protoplanetary ring mean radius $r_m = r + \Delta r/2$. The regions with increased concentration formed by gravitational instabilities accumulate particles from the surrounding space. The second process is the particle attraction to the Sun (the center of the system). Particles that fall on the Sun pull other particles behind them. There is an effect that looks like an avalanche.

Stage 3. The equilibrium stage. The clusters formation stops or the system comes to a homogeneous state at this stage. Generally, many gravitational instabilities are appear, but most of the gravitational instabilities are destroyed during the orbital motion. If the regions with increased concentration are massive, they collapses into a cluster. Clusters interact with each other. Most clusters collide. After such collisions some clusters are disrupted and some clusters are combined. Such collisions continue until the orbits of the clusters become stable.

The result of the third stage depends on the mass of the ring, the radius and the width of the protoplanetary ring. These values determine how many particles will fall on the sun and how many particles will remain in the orbit. If the gravitational instabilities have sufficient mass then they form the clusters, but if they do not have sufficient mass, they break down and their particles scatter in space. Thus in the most cases the clustering can be determined in a relatively short time and long simulations observing many revolutions around the Sun are not needed.

4.2 The Clustering Conditions

More than thousand simulations of the evolution of the protoplanetary rings with different parameters were preformed. The results of all calculations were divided in two classes according to the type of clustering: systems in which clustering is predominantly observed (for analysis of a set of similar systems) and systems in which clustering is predominantly not observed. The simulation results shows that a continuous and smooth boundary between two classes of clustering is observed in the parametric space of $(n/n_0, R_{\max}/R_{\min})$—see Figs. 4 and 5. The figures show that the greater is the concentration of particles in the protoplanetary ring and the greater is the relative width of the ring, the easier clusters formation is initiated. The systems that are close to the boundary can not be assigned to any of the classes with certainty. Such systems have similar probability of both clustering and no clustering. The system position relative to the classes boundary allows to predict the clustering in the system without calculations.

The dependence of the boundary position on the relative mass $M_s = M_{\text{sun}}/M$ was also investigated. The boundary lines were found for $M_s = 5, 10, 20$. The smaller the value of M_s is, the higher is the boundary line (see Fig. 6). These results can be interpreted as follows: the smaller is the influence of the central star, the more likely is the clustering. Gradually, clusters accumulate more and more particles in

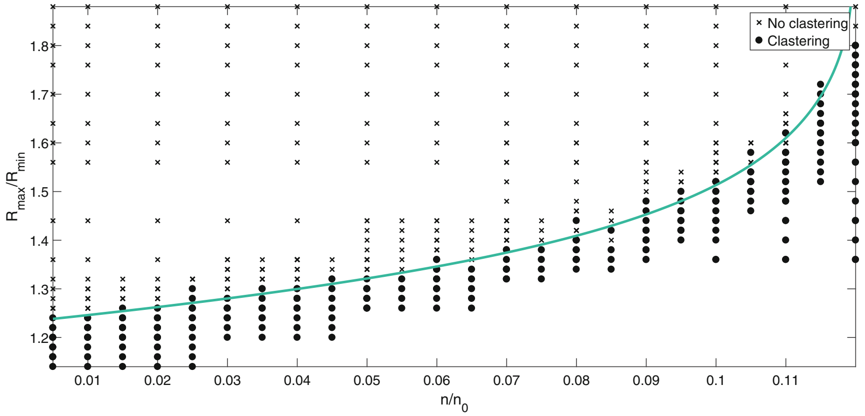

Fig. 4 The clustering diagram. $R_{\min}/a = 200, M_s = 10$

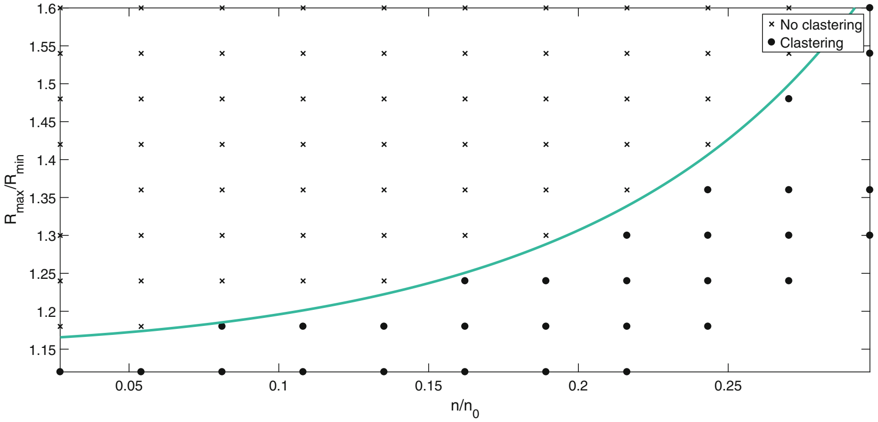

Fig. 5 The clustering diagram. $R_{\min}/a = 200, M_s = 20$

themselves, some clusters merge and some are destroyed. In our experiments, clusters accumulate up to 60% of all system particles (see Fig. 7). Over time, the clusters reach permanent orbits, but the system in which clustering occurred becomes stable usually only when one or two clusters are left in it.

5 Conclusions

The paper studies evolution of the circumsolar (or a circumstellar) rings. The evolution of the rings is simulated by the particle dynamics method. The long-range gravitational forces, the short-range dissipative forces, and the repulsive forces between

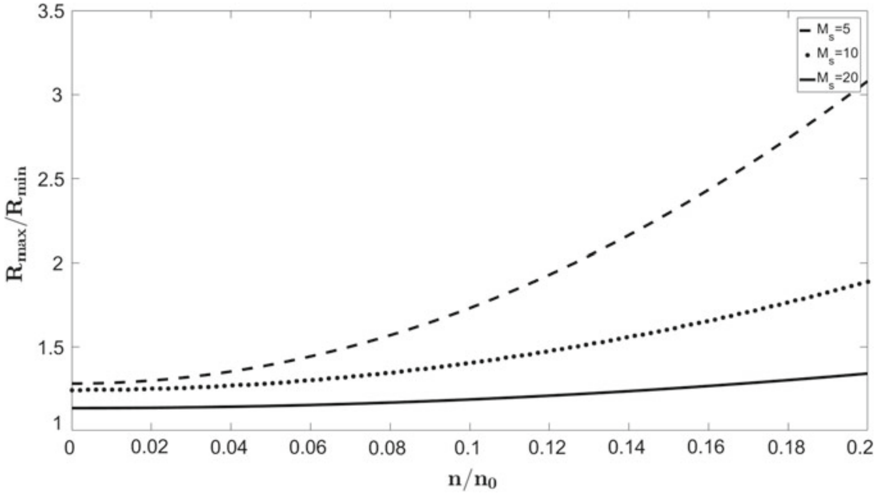

Fig. 6 The classes boundaries for several values of M_s. $R_{\min}/a = 200$

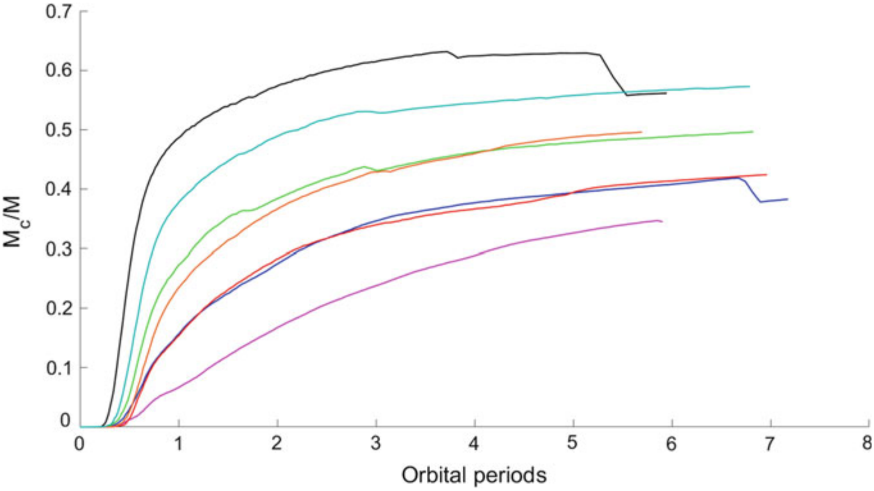

Fig. 7 Change in time the mass ratio of all clusters to the system mass. $R_{\min}/a = 200, M_s = 10$. M_c is the total clusters mass and M is the disk mass. The thresholds on some lines correspond to the clusters destruction by mutual collisions or by falling to the Sun

particles are taken into account. Also, the gravitational force from the central star is applied.

It is demonstrated that there are two main forces affecting the cluster formation in the ring: the self-gravitation of the protoplanetary ring and the gravity of the central star. The first force promotes clustering, the second force hinders it. The balance of these forces determines the formation of clusters. Also, the process of cluster formation is affected by the unique initial distribution of the particles co-ordinates and velocities. Therefore, generally speaking, the possibility of clustering can be estimated only in a statistical sense.

We have found that there is a continuously smooth boundary, which separates systems with and without clustering. The influence of the ring shape, density, and its relative (according to the central star) mass on the boundary position was investigated. The computer modeling shows that in the circumsolar ring the clusters are formed in the regions with an increased concentration of particles. If the concentration of particles is above a certain threshold, then the clusters appear. Then the clusters grow by accumulating particles from the surrounding space. Clusters also interact with each other. Usually after several revolutions only one or two clusters survived, and these clusters have stable orbits.

The further evolution of the clusters has not been studied in the present paper. However previous works [10, 11, 14] have shown that the rotational collapse of a localized gas-dust cloud can lead to the formation of the planet-satellite systems. Such a scenario is possible if the force of the particles interaction can no longer equilibrate the cluster. In particular, the results of the present paper can serve as initial conditions for study of the Earth-Moon formation from a gas-dust cloud, localized on its orbit around the Sun [11].

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