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# The effect of using different rock failure criteria in wellbore stability analysis

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THE EFFECT OF USING DIFFERENT ROCK FAILURE CRITERIA IN WELLBORE STABILITY ANALYSIS

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## ABSTRACT

Determination of the appropriate drilling fluid density by rock failure analysis is an essential step to control wellbore instability. To determine wellbore failure stresses, the rock strength has to be known, an appropriate constitutive model has to be selected, and finally an accurate rock failure criterion must be chosen. Linear-poro elastic modeling of the mechanical wellbore failure is the most common approach to investigate wellbore instability. Numerous failure criteria have been used for the rock failure analysis but there is not any commonly accepted agreement of which failure criteria to use.. Quantitative comparisons have been previously studied on some of the failure criteria but few evaluations for the selection of the failure criteria are based on typical petroleum related situations. In this thesis, the thirteen most common rock failure criteria are compared and analyzed. The rock failure criteria were evaluated for three lithologies. A statistical analysis was performed to determine the similarities and differences among the failure criteria. Field case evaluation of failure criteria was conducted for three different offshore wells. According to the results of the statistical analysis, the Tresca, the Von Mises, and the Inscribed Drucker-Prager represent the higher bounds of results for the minimum required mud weight for all cases. Although the Circumscribed Drucker-Prager usually predicts the lower bound for the minimum required mud weight, its results are in the middle range for hard formations. The Modified Lade, the Modified Wiebols-Cook and the Mogi-Coulomb give similar results for the three cases studied so these failure criteria may be used interchangeably without altering the results. The results of field cases show that the Mogi-Coulomb, the Modified Lade, and the Modified Wiebols-Cook criteria mainly predict minimum required mud weight which is close to the field mud weight used to successfully drill the borehole.

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# NOMENCLATURE

| Symbol | Definition |
|---|---|
| $\sigma_h$ | Minimum Horizontal Stress |
| $\sigma_H$ | Maximum Horizontal Stress |
| $\sigma_V$ | Overburden Stress |
| $\sigma_1$ | Maximum Principal Stress |
| $\sigma_2$ | Intermediate Principal Stress |
| $\sigma_3$ | Minimum Principal Stress |
| $\phi$ | Internal Angle of Friction |
| $C_0$ | Uniaxial Compressive Strength |
| $T_0$ | Uniaxial Tensile Strength |
| $c$ | Cohesion |
| $q$ | Flow Factor |
| $\mu$ | Coefficient of Internal Angle of Friction |
| $\tau$ | Shear Stress |
| $\sigma_{oct}$ | Octahedral Shear Stress |
| $\tau_{oct}$ | Octahedral Shear Stress |
| $\sigma_{m,2}$ | Mean Effective Stress |
| $J_1$ | Mean Effective Confining Stress |
| $J_2$ | Invariant of Deviatoric Stress |
| $I_1$ | First Stress Invariant |
| $I_3$ | Third Stress Invariant |
| $I''_1$ | Modified First Stress Invariant |
| $I''_3$ | Modified Third Stress Invariant |
| $p_w$ | Wellbore Pressure |
| $p_0$ | Pore Pressure |
| ${\sigma_x}^0, {\sigma_y}^0, {\sigma_z}^0, {\tau_{xy}}^0, {\tau_{yz}}^0, {\tau_{xz}}^0$ | Stress Components of Original Wellbore Coordinate System |
| $\sigma_r, \sigma_\theta, \sigma_z, \tau_{r\theta}, \tau_{rz}, \tau_{\theta z}$ | Stress Components of Transformed Wellbore Coordinate System |
| $\sigma_r$ | Radial Stress |
| $\sigma_{t\,min}, \sigma_{t\,max}$ | Maximum and Minimum Tangential Stress |
| $a, b$ | Material Parameters for Mogi-Coulomb Criterion |
| $\eta_1, S$ | Material Parameters for Modified Lade Criterion |
| $\alpha, k$ | Material Parameters for Drucker-Prager Criterion |
| $C_1, A, B, C$ | Material Parameters for Modified Wiebols-Cook Criterion |
| $m, s$ | Material Parameters for Hoek-Brown Criterion |

# ABBREVIATIONS

| Abbreviation | Definition |
|---|---|
| UCS | Uniaxial Compressive Strength |
| CDP | Circumscribed Drucker-Prager |
| ML | Modified Lade |
| MG | Mogi-Coulomb |
| MWC | Modified Wiebols-Cook |
| MR | Murrel |
| HB | Hoek-Brown |
| SD | Stassi D'Alia |
| MC | Mohr-Coulomb |
| MGR | Modified Griffith |
| IDP | Inscribed Drucker-Prager |
| GR | Griffith |
| VM | Von Mises |
| TR | Tresca |
| PD | Percentage Difference |

# 1. INTRODUCTION

Costly consequences of wellbore instability problems are one of the major challenges in drilling operation. Instability is caused when the stresses locally around the wellbore is exceeding the strength of the rock. Drilling fluid pressure can prevent failure to occur if the pressure is kept within the bounds of collapse and fracture gradients around the wellbore. Wellbore stability and pore pressure analysis are the main factors in designing borehole pressure profile for selecting mud density which provides mechanical stability of the borehole wall and decrease Non-Productive Time (NPT). The wellbore stability model must include rock strength properties which governs the behavior of the rock when it is subjected to the in-situ stresses. A rock failure criterion specifies stress condition at failure. Therefore to determine wellbore failure stresses, the rock strength has to be known, an appropriate constitutive model has to be selected, and finally an accurate rock failure criterion must be chosen. Linear-poro elastic modeling of the mechanical wellbore failure is the most common approach to investigate wellbore instability. For safe well construction, in-situ stresses must be used to predict minimum required mud weight which cause break outs of the wellbore wall.

## 1.1. OVERVIEW ON WELLBORE STABILITY ANALYSIS

Over the last 30 years there have been numerous investigations into developing analytical models for mechanical wellbore stability (Bradley, 1979, Aadnoy, 1987, McLean and Addis, 1990). All models have in common that they consider the wellbore as a circular hole in a rigid infinite body subjected to far field principal compressive stresses. The common form is to consider the wellbore constitutive model as a linear poro-elastic material with isotropic rock deformation and strength properties in a 3D principal stress field. The minimum drilling fluid pressure is determined to prevent the differential stresses around the wellbore to be less than the shear strength of the rock. Different rock failure criteria were used for wellbore stability analysis to predict onset of wellbore failure but there is no agreement on which failure criterion to be used for wellbore stability analysis. Based on the different characteristics of the rock failure criteria, their result for the minimum drilling fluid density might be significantly different. It is mainly due to how rock failure criteria include the effect of in-situ principal stresses. The estimation of minimum drilling fluid density by different criteria may lead to unsafe or conservative approach in the result of wellbore stability analysis.

## 1.2. MOTIVATION FOR EVALUATION OF ROCK FAILURE CRITERIA

Previous studies on the evaluation of rock failure criteria is mainly focused on the quantitative comparison or determination of the best fitting parameters for the different rock failure criteria based on triaxial test results data. Reviewing previous studies reveals that some failure criteria, including Stassi d'Alia, have not been considered (Mclean and Addis, 1990, Song and Haimson, 1997, Ewy, 1999, Colmenares and Zoback, 2002, Yi et al., 2005, Al-Ajmi and Zimmerman, 2006, Simangunsong et al., 2006, Nawrocki, 2010, Zhang et al., 2010). Also in some of the previous studies, hypothetical data set have been used for the stress data, rock mechanical properties, and well depth which caused results to be unrealistic in some cases. For instance, true vertical well depth in the range of 12,000 m to 28,000 m has been chosen for analysis which caused the results not to be directly applicable to the stability of wells for petroleum exploitation (Zhang et al., 2010). Furthermore, quantitative comparisons have been previously studied on some of failure criteria but few evaluations of the failure criteria are based on typical petroleum related situations.

## 1.3. RESEARCH OBJECTIVES

The overall objective of this study is to perform a statistical comparison to investigate similarities and differences of various rock failure criteria for prediction of the minimum drilling fluid density under different rock lithology and stress data. Thirteen of the most common rock failure criteria which are used for wellbore stability analysis have been statistically evaluated for three rock lithologies including shale, sandstone, and siltstone. Rock mechanical and stress data from the Rulison field in western Colorado was used (Higgins, 2006). Results of the statistical analysis are presented using percentage difference method and table of contradictions. The field case evaluation of failure criteria was done for three different offshore wells in North Sea and Indonesia. Estimated minimum required mud weight by different criteria compared to the actual mud weight used to successfully drill the borehole. The thesis is based on a situation where there is overbalance in the wellbore (i.e. mud pressure is higher than the in-situ formation fluid pressure).

# 2. LITERATURE REVIEW

In-situ stresses and rock mechanical properties are key input data for wellbore stability analysis. In-situ stresses can be defined by three principal stresses that determine the loading level on the rock. To determine in-situ stresses on borehole wall, an appropriate constitutive model should be chosen. The linear elasticity theory for solid material does not give accurate description of rock under loading due to the effect of pore pressure, so linear poro-elasticity theory is the common approach in petroleum related geomechanics (Fjaer et al., 2008). The rock failure condition should be known to find the minimum required mud weight for keeping the mechanical stability of wellbore wall. Failure criteria were developed to estimate stress concentration at which rock fails. Prediction of accurate stress condition at which rock breakouts occurs is one of the main challenges which standout in wellbore stability analysis.

## 2.1. ROCK FAILURE CRITERIA

A failure criterion specifies at which stress condition failure occurs. The application of failure criteria would be limited to the soil, rock, or metal based on their features. In wellbore stability analysis, the rock failure criterion is used to determine borehole breakout pressure due to tensile, shear or compressive failure. Numerous rock failure criteria are proposed with different characteristics. Earlier developed rock failure criteria are two dimensional since the effect of intermediate principal stresses was not known. Since the effect of intermediate principal stress is relatively low compared to the minimum and maximum principal stresses, still two dimensional failure criteria will be applicable. Some of the failure criteria were included intermediate principal stress by using rotational symmetry in three dimensional stress space but there was not any experimental evidence for that inclusion. Based on the original true triaxial test results by Mogi (1971) and the experimental analysis done later, the influence of intermediate principal stress on the rock strength has been verified (Takahashi and Koide, 1989, Al-Ajmi and Zimmerman, 2005). As the stress state around the borehole wall is true triaxial, failure criteria which accounts for the intermediate principal stress possibly give the accurate stress condition of failure. Linearity is the second characteristic which failure criteria will be categorized based on. Non-linear form of failure criteria have been developed to give better curve fitting with polyaxial test results data. In the principal stresses space, rock failure criteria will be identified by cross sectional shape of failure surface. In the following section each individual failure criteria will be evaluated. Since failure criteria are mainly derived based on the development or modification of previous failure criteria, the failure criteria are presented in this section in the order when they first appeared in the literature.

**2.1.1. Mohr-Coulomb.** The Mohr-Coulomb is the most commonly used failure criterion in geomechanics and has a simple linear form. Mohr-Coulomb criterion is based on the two dimensional Mohr's stress circle which is useful theory in analyzing rock failure. Coulomb concluded rock failure will occur alongside a plane due to acting shear stress on that plane. Failure of a plane will be resisted by frictional force which is function of internal friction of rock, the normal stress components and also internal cohesion of rock. According to Coulomb's failure theory, required compressive stress for failure will be increase linearly by increasing the confining stress. Rock failure will occur when following condition is met (Derivation of equation 2.1 is shown in appendix A):

$$\tau = c + \mu\sigma \tag{2.1}$$

The parameter $\boldsymbol{c}$ is known as cohesion and parameter $\boldsymbol{\mu}$ is the coefficient of internal angle of friction ($\phi$). The tangential point of Mohr circle of principal in-situ stresses and Mohr-Coulomb failure envelope in $\tau - \sigma$ space represents state of stress which rock will fail in shear. The coefficient of internal angle of friction will be defined with following relation:

$$\mu = \tan\phi \tag{2.2}$$

The Mohr-Coulomb failure criterion can be expressed in term of maximum and minimum principal stresses (Derivation of equation 2.3 is shown in appendix A).

$$\sigma_1 = \mathrm{C}_0 + \sigma_3 q \tag{2.3}$$

The $C_0$ is uniaxial compressive strength (UCS) which is related to cohesion of rock and internal angle of friction. The parameter $q$ is the flow factor which is function of the internal angle of friction.

$$C_0 = 2c\frac{\cos\phi}{1-\sin\phi} \tag{2.4}$$

$$q = \frac{1+\sin\phi}{1-\sin\phi} \tag{2.5}$$

These two major rock mechanical properties ($C_0$ , $q$) have been considered as basic input data for the evaluation of material parameters for different failure criteria considered. Mohr-Coulomb's failure criterion does not consider the effect of intermediate principal stress on the rock strength which is in contrast to the true triaxial state of stress in rock.

**2.1.2. Mogi-Coulomb.** Mogi (1971) conducted true-triaxial experiments on different rock types and interpreted the results. Based on the test results, Mogi concluded the intermediate principal stress ($\sigma_2$) influences the rock strength for several rock lithologies and the fracture occurs along a plane in the direction of intermediate principal stress. Based on the polyaxial test results, Mogi pointed out that mean normal stress which opposes creation of fracture is $\sigma_{m,2}$, rather than the octahedral normal stress, $\sigma_{oct}$. His observations from the laboratory tests lead to a rock failure criterion which takes into account the effect of intermediate principal stress. Mogi formulated the assumption that, distortional strain energy as a frictional force is proportional to the octahedral shear stress, $\tau_{oct}$ and will be increased by increasing $\sigma_{m,2}$ until a critical level where failure occur. Based on this result, Al-Ajmi and Zimmerman (2005) found a linear relation that could fit well with polyaxial tests data in $\tau_{oct}$-$\sigma_{m,2}$ space:

$$\tau_{oct} = a + b\sigma_{m,2} \tag{2.6}$$

The parameter $a$ is the intersection of line on the $\tau_{oct}$ axis and $b$ is the inclination of the line. Al-Ajmi and Zimmerman (2005) tested the developed linear function for the polyaxial test data from eight different rock types and the results show that Mogi-Coulomb criterion gives a good fit to polyaxial tests data. They also found out this polyaxial criterion correlates well with conventional triaxial test data. Al-Ajmi and Zimmerman (2005) presented the relation between the linear Mogi-Coulomb failure criterion parameters $a$ and$b$ based on the Mohr-Coulomb failure criterion parameters, $q$ and $C_0$ :

$$a = \frac{2\sqrt{2}}{3}\frac{C_0}{q+1} \tag{2.7}$$

$$b = \frac{2\sqrt{2}}{3}\frac{q-1}{q+1} \tag{2.8}$$

Since linear Mogi-Coulomb failure criterion is equivalent to Mohr-Coulomb in conventional triaxial stress state, it can be seen as an extension of Mohr-Coulomb in true triaxial space.

**2.1.3. Tresca.** Shear failure occurs when shear stress along some plane reaches a critical level. Tresca is the simplest possible criterion for shear failure based on the Mohr's stress circle. Tresca (1864) assumed that failure would occur if maximum shear failure inside any planes of rock reaches a critical value which is equal to cohesion or intrinsic shear strength of rock.

$$\frac{(\sigma_1-\sigma_3)}{2} = \tau_{max} = c \tag{2.9}$$

$$\frac{\mathrm{C}_0}{2} = c \tag{2.10}$$

Where the parameter $c$ is cohesion of rock and $\mathrm{C}_0$ is the uniaxial compressive strength (UCS). Maximum shear stress is derived from Mohr's stress circle. In the three dimensional principal stresses space, cross section of Tresca criterion is like regular hexagon. Tresca can be considered as especial case of Mohr-Coulomb failure criteria when internal angle of friction is equal to zero. Recall equations 2.3, 2.4 and 2.5, if the internal angle of friction set as zero, Mohr-Coulomb criterion reduced to following form which is Tresca criterion:

$$\sigma_1 = \mathrm{C}_0 + \sigma_3 q$$

$$\text{when } \phi = 0 \quad q = \frac{1 + \sin\phi}{1 - \sin\phi} = 1$$

$$\text{and} \quad C_0 = (\sigma_1 - \sigma_3) \tag{2.11}$$

**2.1.4. Von Mises.** Before development of true triaxial experiment, couple of scientists tried to include the effect of intermediate principal stress in failure criterion. Although without any experimental evidence, Von Mises (1913) proposed his failure criterion in an interesting way by using rotational symmetry of $\pi$ –plane which leads to a circular cross sectional shape of failure surface in three dimensional stress space. He assumed that rock fails when square root of the second invariant of deviatoric stress $(J_2)$ reaches a critical level. Based on the different form of $(J_2)$, Von Mises failure criterion can be written in following forms:

$$\sqrt{J_2} = \frac{C_0}{3} \tag{2.12}$$

$$\sqrt{J_2} = \sqrt{\frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right]} = \frac{C_0}{3} \tag{2.13}$$

The Von Mises criterion was developed for metals and it has very limited application for rocks since it defines failure mechanism which is independent of stress magnitude of material and it is in contrast with experimental observation since yield of many rocks increases with increasing mean normal stress. This idea was used later for the development of a failure criterion that includes the mean normal stress component.

**2.1.5. Drucker-Prager.** Drucker and Prager proposed their failure criterion in the sense to extend application of Von-Mises failure criterion to the rock mechanics (Drucker and Prager, 1952). As it has been mentioned, Von Mises failure criterion developed without experimental foundation and just by using the rotational symmetry of $\pi$-plane in three dimensional stress spaces. The yield stress in most rock types will increase by increasing mean normal stress but Von Mises did not include mean normal stress component. Therefore, Von Mises failure criterion does not fit experimental observation. Originally developed for soil mechanic and as an extension of Von Mises criterion, Drucker and Prager proposed new failure criterion by including the mean normal stress component, $J_1$ (Drucker and Prager, 1952). The yield surface of the modified Von Mises criterion in principal stresses space is a right circular cone equally inclined to the principal-stress axes. The intersection of the π–plane with this yield surface is a circle.

$$\sqrt{J_2} = k + \alpha J_1 \tag{2.14}$$

The parameter $k$ and $\alpha$ are material constants and $J_1$ is the mean effective confining stress:

$$J_1 = \frac{1}{3}\,(\sigma_1 + \sigma_2 + \sigma_3) \tag{2.15}$$

The material parameters $\alpha$ and $k$ can be identified from the slope and the intercept of the failure line which has been plotted in the $J_1$ and $\sqrt{J_2}$ space. Since the parameter $\alpha$ is related to the internal friction of the rock and $k$ is related to the cohesion of rock. Mohr-Coulomb criterion parameters could be used to determine Drucker-Prager failure criterion parameters. Based on the fitting triaxial test data and comparison with the Mohr-Coulomb criterion in three dimensional stress spaces, the Drucker-Prager criterion can be divided into Circumscribed Drucker-Prager and Inscribed Drucker-Prager. The outer Drucker Prager or circumscribed coincides with the outer apices of the Mohr-Coulomb hexagon in the cross sectional view of π-plane while the inscribed Drucker-Prager touches the inside of Mohr-Coulomb hexagon. It should be noted that inscribed Drucker Prager is the result of trigonometric fitting between Mohr-Coulomb and Drucker-Prager criteria and actually does not fit the triaxial test data (Mclean and Addis, 1990). For the Inscribed Drucker-Prager, material parameters would be (Vekeens and Walters, 1989):

$$\alpha = \frac{3\sin\phi}{\sqrt{9+3\sin\phi^2}} \tag{2.16}$$

$$k = \frac{3C_0\cos\phi}{2\sqrt{q}\sqrt{9+3\sin\phi^2}} \tag{2.17}$$

For the Circumscribed Drucker-Prager, following solution has been presented using Mohr-Coulomb criterion parameters (Zhou 1994):

$$\alpha = \frac{\sqrt{3}(q-1)}{(2+q)} \tag{2.18}$$

$$k = \frac{\sqrt{3}C_0}{2+q} \tag{2.19}$$

Main shortcoming of Drucker-Prager criterion is giving same weight to intermediate principal stress as minimum and maximum principal stresses while we know the intermediate principal stress does effect on the rock strength but it is not significant as other two principal stresses. That is the main reason why circumscribed Drucker-Prager typically represents the lowest bound of results for the minimum required mud weight.

**2.1.6. Modified Wiebols-Cook.** Wiebols and Cook (1968) proposed a model which describes the impact of the intermediate principal stress ($\sigma_2$) on rock strength. By considering shear strain energy of microcracks in the rock, they gave physical description of sliding microcracks surfaces which cause failure when the stress condition meet frictional criterion. In better words, by micromechanical analysis of sliding cracks, Wiebols and Cook concluded the rock will fail when shear strain energy which is enclosed with microcracks reaches a critical level. A model which presented later on (Zhou, 1994) as an extension of circumscribed Drucker Prager is a nonlinear criterion which is called Modified Wiebols-Cook due to similarities to the original model by Wiebols and Cook (Zhou, 1994). Here is the criterion presented by Zhou:

$$\sqrt{J_2} = A + BJ_1 + CJ_1{}^2 \tag{2.20}$$

Where $J_1$ is the mean effective confining stress and $J_2$ is the second invariant of deviatoric stress related to octahedral shear stress. The mean normal stress component in Modified Wiebols-Cook has quadratic form while in the Drucker-Prager criterion has linear form. The parameters A, B, and C can be determined from result of conventional triaxial test under different conditions. The Mohr-Coulomb criterion parameters including uniaxial compressive strength ($C_0$) and flow factor ($q$) can be used as input for determination of *A*, *B*, and *C* parameters as shown here:

$$C = \frac{\sqrt{27}}{2C_1+(q-1)\sigma_3-C_0}\left(\frac{C_1+(q-1)\sigma_3-C_0}{2C_1+(2q-1)\sigma_3-C_0} - \frac{q-1}{q+2}\right) \tag{2.21}$$

The parameter $q$ is the flow factor as defined in relation 2.5 and $C_1$ is function of internal angle of friction and uniaxial compressive strength($C_0$):

$$C_1 = (1 + 0.6\,\mu_i)C_0 \tag{2.22}$$

Where $\mu_i$ the coefficient of internal is frictional angle and presented.

$$B = \frac{\sqrt{3}(q-1)}{q+2} - \frac{C}{3}[2C_0 + (q+2)\sigma_3] \tag{2.23}$$

And parameter *A* is function of *B* and *C*:

$$A = \frac{C_0}{\sqrt{3}} - \frac{C_0}{3}B - \frac{{C_0}^2}{9}C \tag{2.24}$$

**2.1.7. Hoek-Brown.** Hoek and Brown (1980) developed an empirical model for the failure of fractured rock. To investigate the failure of fracture rocks, both fracture properties and rock properties should be taken into account. As result of existing fractures and lower resistance for the shear failure, the fractured rock is weaker than intact rock. Failure criterion which has been proposed by Hoek and Brown included both rock and fracture properties:

$$\sigma_1 = \sigma_3 + \sqrt{mC_0\sigma_3 + sC_0{}^2} \tag{2.25}$$

Where the uniaxial compressive strength, $C_0$ is function of rock properties while $m$ and $s$ are constant depending on the both rock properties and fracture characteristics. For the intact rock, material constant, $s$ is equal to 1 and the fractured rock will be categorized based on "1- $s$" relation. Although there is a relation between material constant $m$ and internal angle of friction but still no mathematical relation has been presented for that (Fjaer et al., 2008). This could be count as a disadvantage of Hoek-Brown failure criterion since the material constant $m$ could not be evaluated based on the experimental data or well logging data . Therefore in same practical situation, Mohr-Coulomb will be preferred since the criterion' constants can be identified by basic rock mechanical data from triaxial tests results. Range of the material constant $m$ is in interval of 5 to 30 depends on the rock types (Zoback, 2007). Since empirical Hoek and Brown failure criterion follow the Mohr's stress circle, it does not consider the effect of intermediate principal stress. Predicted ratio of compressive to tensile strength by Hoek-Brown criterion is larger than what the Mohr-Coulomb criterion does, in this sense Hoek-Brown gives closer agreement to experimental observation (Jaeger et al., 2007). Empirical Hoek-Brown failure criterion has nonlinear form in contrast to the linear Mohr-Coulomb criterion.

**2.1.8. Modified Lade.** According to experimental observations, Lade (1977) concluded that for cohesionless soil, the frictional angle decreases with increasing the magnitude of mean normal stress. An original relation which was developed by Lade in term of first and third stress invariants includes material constant and atmospheric pressure parameters. The stress invariant parameters, $I_1$ and $I_3$ in Lade criterion did not determine based on the effective stresses concept. The Modified Lade criterion was developed by Ewy (1999). In Ewy's version of Lade criterion, material constant, $m$ has been considered as zero in order to modify the original lade criterion in the sense that linear shear strength increases with increasing the first stress invariant, $I_1$ or mean normal stress invariant $I_1/3$ . Furthermore, since original Lade criterion is defined for the cohesionless material, Ewy proposed new way with effective stresses and also introduction of the parameter $S$ as a function of cohesion which changed the Lade criterion to following form:

$$\frac{I_1^{"3}}{I_3^{"}} = 27 + \eta \tag{2.26}$$

Where the appropriate stress invariants $I_1^{"}$ and $I_3^{"}$ express as:

$$I_1^{"} = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \tag{2.27}$$

$$I_3^{"} = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S) \tag{2.28}$$

The Modified Lade criterion has two main advantages. First, it does include the effect of intermediate principal stress on the rock strength. The second advantage is that parameters $S$ and $\eta$ can be determined by Mohr-Coulomb criterion parameters including cohesion and internal angle of friction which can be evaluated based on the triaxial test data or well logging data:

$$S = \frac{c}{\tan\phi} \tag{2.29}$$

$$\eta = \frac{4\tan\phi^2(9-7\sin\phi)}{(1-\sin\phi)} \tag{2.30}$$

By introducing the parameter $S$ in the stress invariant components, the application of Modified Lade criterion can be extended to rock mechanical applications. In the three dimensional stress space, Modified Lade criterion has an triangular cone shape or convex which will slightly change depends on the value of $\eta$. The Modified Lade criterion estimates that shear strength will be increased by increasing mean effective stress as a frictional force. Since Modified Lade has no tension cut off component, it is not accurate in presence of tensile stress. This is not a concern in a wellbore stability analysis because the interest points on the borehole wall which required mud weight for state of balance are not subjected to tension (Ewy, 1999).

**2.1.9. Griffith.** The analysis of microcracks in a two dimensional model was Griffith's starting point for developing a failure criterion which can be applied under both tensile and compressive conditions (Griffith, 1921, Jaeger et al., 2007). Expansion of microcracks is a function of the tensile stress at the tip of the crack if it exceeds a critical level. Compressive stress may cause expansion of microcracks if in an anisotropic stress state. The stress condition at tip of crack becomes tensile due to the orientation of stresses at skew angle related to maximum principal stress (Fjaer et al., 2008). Based on this assumption, Griffith applied his failure theory to general stress state and original Griffith criterion was developed in a nonlinear form in Mohr's space and in terms of uniaxial tensile strength, $T_0$ and two dimensional stress state components:

$$(\sigma_1 - \sigma_3)^2 = 8T_0(\sigma_1 + \sigma_3) \tag{2.31}$$
$$\sigma_3 = -T_0 \qquad \text{if} \quad \sigma_1 + 3\sigma_3 < 0$$

$$T_0 = \frac{C_0}{8} \tag{2.32}$$

In σ-τ plane, Griffith criterion can be expressed in the following form:

$$\tau^2 = 4T_0(\sigma + T_0) \tag{2.33}$$

The Griffith criterion has a parabola curve in two dimensional principal stress space which is steeper in low confining stress and close to straight line in high confining stress. The constant ratio of uniaxial compressive strength to uniaxial tensile strength which was presented by Griffith seems to be practical since it is lower, but close to typical experimental observation for the range of this ratio, 10 to 15 (Fjaer et al., 2008). One of the disadvantages of this failure criterion is its dependence on a single variable which makes it harder to fit both conventional triaxial and polyaxial test data. The second shortcoming of the Griffith criterion is lack of considering the effect which intermediate principal stress has on the rock strength. However, the shape of Griffith criterion on the three dimensional stress space can be described using the symmetry same as Mohr-Coulomb failure criterion. As matter of mentioned fact, Griffith criterion will end up with a regular hexagonal shape of cross section of failure surface in three dimensional stress space.

**2.1.10. Modified Griffith.** Griffith criterion has parabola curve in two dimensional principal stress spaces which is steeper in low confining stress and close to straight line in high confining stress. Based on the experimental observation, Griffith criterion may give a fairly good description of failure at low confining stresses, while Mohr–Coulomb criterion gives a better description of failure at higher confining stresses by straight line. Under compression, shear failure due to the closure of crack can occur before tensile stress reaches a critical level at the tip of crack to initiate fracture; therefore, criterion should be developed in a way to include shear mechanism and frictional behavior (Brace, 1960). McClintock and Walsh (1962) included the effect of friction between crack faces and Modified Griffith was presented. Ratio of uniaxial compressive strength to uniaxial tensile strength is a function of coefficient of internal angle of friction:

$$\frac{C_0}{T_0} = \frac{4}{\sqrt{\mu^2+1}-\mu} \tag{2.34}$$

It should be noted that the required compressive stress to close the crack has been considered to be small. By neglecting the stress required to close cracks, the Modified Griffith criterion can be reduced to the following form:

$$\sigma_1\left[\sqrt{\mu^2+1}-\mu\right]-\sigma_3\left[\sqrt{\mu^2+1}+\mu\right]=4T_0 \tag{2.35}$$

Modified Griffith criterion is similar to the Mohr-Coulomb criterion if we consider the intrinsic shear strength can be evaluated by setting normal stress component as zero in relation 2.33 which results in $\tau = 2T_0 = c$. Modified Griffith does not consider influence of intermediated principal stress and has been modeled in two dimensional stress space. Although, investigation of the rock failure mechanism with expansion of single crack is qualitatively a great way for understanding of failure as function of the stress state but at the same time it is the oversimplification of complicated processes in rock failure and deformation (Jaeger et al., 2007). Computational methods like finite element analysis have been used lately to give more realistic explanation of failure based on the micromechanical analysis of rock (Jaeger et al., 2007).

**2.1.11. Murrel.** Extended Griffith criterion was introduced by Murrel (1963) and includes intermediate principal stress as contributing factor in the rock strength. Murrel extended Griffith to three dimensional stress spaces:

$$(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = 24T_0(\sigma_1 + \sigma_2 + \sigma_3) \qquad (2.36)$$

The ratio of uniaxial compressive strength to uniaxial tensile strength which was presented by Murrel is close to the typical range of experimental observation (Fjaer et al., 2008):

$$T_0 = \frac{C_0}{12} \qquad (2.37)$$

In term of octahedral stresses, the Murrel criterion can be written in the following form:

$$\tau^2{}_{oct} = 4T_0\sigma_{oct} \qquad (2.38)$$

As mentioned for the Griffith and the Modified Griffith criterion, investigation of the rock failure mechanism as an expanding single crack is qualitatively a good way to understand failure as function of stress state it oversimplifies the complicated processes of rock failure and deformation (Jaeger et al., 2007). The cross section of the predicted failure surface by the Murrel criterion has a circular shape in the three dimensional stress domain.

**2.1.12. Stassi D'Alia.** By modifying the plastification condition, Stassi d'Alia (1967) developed a non-linear failure criterion in terms of principal stresses and uniaxial tensile strength:

$$(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = 2(C_0 - T_0)(\sigma_1 + \sigma_2 + \sigma_3) + 2C_0T_0 \qquad (2.39)$$

Since the increasing tensile component might have a peculiar effect on the results of the minimum required mud weight by the Stassi d'Alia criterion, commonly the tensile component is set equal to zero (Stjern et al., 2000). Here in this study, the McClintock and Walsh (1962) assumption were used. McClintock and Walsh assumed that the tensile component in Eq. 39 could be determined by the Modified Griffith criterion ratio of uniaxial compressive strength to uniaxial tensile strength (Eq. 2.34).

## 2.2. BOREHOLE STRESS TRANSFORMATION

The excavation of the underground formation causes redistribution of the stress state and the drilling fluid should have sufficient density to provide the mechanical stability of the borehole wall. In-situ principal stresses have to be known to determine the appropriate drilling fluid density to keep borehole stable. Therefore, stress determination around the borehole is a key step in wellbore stability analysis. A linear poro-elasticity model is the most common approach for wellbore stability analysis. The stress components of the wellbore wall in a linear poro-elasticity model are given as (Bradley, 1979, Aadnoy and Chenevert, 1987):

$$\sigma_r = p_w \qquad (2.40)$$

$$\sigma_\theta = \sigma_x{}^0 + \sigma_y{}^0 - 2\left(\sigma_x{}^0 - \sigma_y{}^0\right)\cos 2\theta - \tau_{xy}{}^0 \sin 2\theta - p_w \qquad (2.41)$$

$$\sigma_z = \sigma_z{}^0 - \upsilon\left[2\left(\sigma_x{}^0 - \sigma_y{}^0\right)\cos 2\theta + 4\tau_{xy}{}^0 \sin 2\theta\right] \qquad (2.42)$$

$$\tau_{r\theta} = 0 \qquad (2.43)$$

$$\tau_{\theta z} = 2\left(-\tau_{xz}{}^0 \sin\theta + \tau_{yz}{}^0 \cos\theta\right) \qquad (2.44)$$

$$\tau_{rz} = 0 \qquad (2.45)$$

$p_w$ is the drilling fluid pressure, ${\sigma_x}^0, {\sigma_y}^0, {\sigma_z}^0, {\tau_{xy}}^0$, ${\tau_{yz}}^0$, and ${\tau_{xz}}^0$ are stress components of original coordinate system. Since the shear stress component are non-zero, $\sigma_r$, $\sigma_\theta$, and $\sigma_z$ are not principal stresses. Principal stresses can be determined by following formula:

$$\sigma_r{}' = p_w - p_0 \tag{2.46}$$

$$\sigma_{t\,max} = \frac{1}{2}\left((\sigma_z + \sigma_\theta) + \sqrt{(\sigma_z - \sigma_\theta)^2 + 4{\tau_{\theta z}}^2}\right) - p_0 \tag{2.47}$$

$$\sigma_{t\,min} = \frac{1}{2}\left((\sigma_z + \sigma_\theta) - \sqrt{(\sigma_z - \sigma_\theta)^2 + 4{\tau_{\theta z}}^2}\right) - p_0 \tag{2.48}$$

The maximum and minimum tangential stresses which are perpendicular to each other are denoted by $\sigma_{t\,max}$ and $\sigma_{t\,min}$ . The effective radial stress is one of the other principal stress component denoted by $\sigma_r{}'$. This procedure should be repeated for different orientation in cylindrical coordinate system around the wellbore in order to find maximum differential stress at the wellbore wall.

## 2.3. NUMERICAL SOLUTION FOR DETERMINATION OF MINIMUM REQUIRED MUD WEIGHT

Determination of borehole principal stress components is the first step in identifying the minimum required mud weight. The linear poro-elasticity model as the most common approach for a wellbore stability analysis was chosen to determine in-situ principal stresses. The borehole in-situ principal stresses which have been determined by the linear poro-elasticity model are used to calculate the minimum drilling fluid density by the rock failure criterion. Iterations are required to determine breakout pressure (Fjaer et al., 2008).

## 2.4. LITERATURE REVIEW ON THE EVALUATION OF ROCK FAILURE CRITERIA

One of the first evaluations of rock failure criteria was done by Mclean and Addis (1990). They compared Mohr-Coulomb and different forms of Drucker-Prager for the prediction of the minimum mud weight. The linear elasticity borehole stability model was used in their analysis. Failure criteria were evaluated for a sandstone formation in North Sea. Based on the results, they concluded that the Inscribed Drucker-Prager conservatively predicts the stresses condition at failure which results in higher minimum required mud weight compared to the Mohr-Coulomb and the Circumscribed Drucker-Prager criteria. The Inscribed Drucker-Prager and the Mohr-Coulomb estimated similar results for the vertical borehole but by increasing borehole deviation the Inscribed Drucker-Prager predicts much higher minimum required mud weight than the Mohr-Coulomb criterion. Minimum mud weight predicted by the Circumscribed Drucker-Prager was lower than the result of the Mohr-Coulomb and the Inscribed Drucker-Prager criteria. This difference was significant for the vertical borehole compared to the horizontal borehole where this difference is lower. In their reported field case a horizontal well was drilled successfully with a mud weight close to the Circumscribed Drucker-Prager mud weight predictions rather than the Mohr-Coulomb and the Inscribed Drucker-Prager. In other case, they analyzed the results of required overbalance for drilling a vertical well in the weak sand formation by those three failure criteria. Although overbalance was required for drilling the weak sand formation and should be increased by depth, the Circumscribed Drucker-Prager estimated that no overbalance was required for drilling the weak sand formation regardless of depth. Due to the major difference of mud weight predictions by the Circumscribed Drucker-Prager in two case studies, Mclean and Addiss concluded that a criterion can predict realistic result in one situation but give unrealistic results for other conditions. The Mohr-Coulomb failure criterion was recommended for wellbore stability analysis because of the more realistic results compared to the results from the different forms of Drucker-Prager (Mclean and Addis, 1990).

Song and Haimson (1997) did laboratory simulations of borehole breakouts and triaxial tests on Westerly granite and Berea sandstone to determine an appropriate criterion for estimating breakout failure for each rock. They concluded that the polyaxial Mogi criterion, which includes the intermediate principal stress, can estimate the stress condition for breakout failure in a close agreement with breakout boundaries found in the laboratory tests for both rock types. The Mohr-Coulomb criterion did not accurately predict breakout in these two types of rocks compared to the polyaxial Mogi criterion. They found the laboratory experiment of breakout failure as good method to determine appropriate failure criteria for a given rock type.

The Modified Lade criterion was developed by Ewy (1999) as a polyaxial failure criterion which is compatible with the triaxial stress state in the rock. The results of the predicted minimum required mud weight by the Modified Lade have been compared with two common rock failure criteria, the Mohr-Coulomb and the Circumscribed Drucker-Prager. Ewy used linear elasticity model to determine in-situ principal stress components on the borehole wall. The rock failure criteria were evaluated on three rock lithologies including sandstone, shale and poorly cemented sand. The typical range of rock mechanical properties used for these three lithologies and a hypothetical data set was considered for in-situ stresses. According to the results, there was a major difference between the results of Modified Lade and the other two criteria, Mohr-Coulomb and Drucker-Prager. The predicted minimum required mud weight by the Modified Lade is not that high as the Mohr-Coulomb and it is not low same as once predicted by the Drucker-Prager. Furthermore, difference between the results of minimum required mud weight from the vertical to horizontal borehole for Modified Lade is not significant as the Mohr-Coulomb and the Drucker-Prager. Ewy concluded this might be the results of including intermediate principal stress in a right way.

Colmenares and Zoback (2002) evaluated seven different rock failure criteria based on the fitting of polyaxial test data for five different rock types. They concluded that the Modified Wiebols-Cook and the Modified Lade failure criteria had best fit with polyaxial test data, especially for those rocks which their failure behavior are significantly depend on the intermediate principal stress. According to their results, for rocks which are not dependent on the intermediate principal stress, the Mohr-Coulomb and the Hoek-Brown criteria had good fit with the polyaxial test data even better than the complicated polyaxial criteria. None of the Inscribed or the Circumscribed Drucker-Prager gave a close fit with the polyaxial test data for the five different rock types. They also found out that the polyaxial failure criteria might give a close fit to the conventional triaxial test data and also incorporates the effect of the intermediate principal stress. It was concluded that the Mohr-Coulomb criterion underestimates rock strength and the Drucker-Prager criterion overestimates rock strength. Using of the Modified Wiebols-Cook and the Modified Lade failure criteria was recommended.

Yi et al. (2005) compared three common rock failure criteria based on the minimum mud weight estimation by using polyaxial test data for Yuubari shale and Dunham dolomite. The linear poro-elasticity model was used to determine borehole in-situ stresses. The Mohr-Coulomb, the Drucker-Prager and the Modified Lade were the three failure criteria evaluated in their study. A hypothetical data set for in-situ stress and pore pressure was used for the evaluation of rock failure criteria. They concluded that failure criterion which fits the polyaxial test data best describes the rock failure better, and therefore gives more reliable results for the minimum required mud weight predictions. Based on their results, no particular failure criterion estimates higher or lower minimum mud weight for all the cases studied. For Dunham dolomite, the Modified Lade criterion has good fit with polyaxial test data and therefore, predicted results of minimum required mud weight by the Modified Lade supposed to be reliable. In case of Yuubari shale, the Mohr-Coulomb criterion has good fit with polyaxial test data and estimated results of minimum required mud by the Mohr-Coulomb criterion are considered to be reliable.

Al-Ajmi and Zimmerman (2006) developed the linear form of Mogi-Coulomb and compared that with the Mohr-Coulomb failure criterion. They proposed the use of the Mogi-Coulomb over the Mohr-Coulomb regarding both fitting polyaxial test data and also prediction of the borehole breakout pressure. Since the Mogi-Coulomb can properly account strengthening effect of intermediate principal stress, it can be reliable criterion for the estimation of borehole breakout pressure compare to the Mohr-Coulomb criterion. The field case evaluation of failure criteria was done on four offshore wells. The results of field case studies verified that the Mogi-Coulomb estimates more realistic results for the minimum required mud weight than the Mohr-Coulomb criterion. They found out that borehole stability is remarkably dependent on the intermediate principal stress and using polyaxial failure criteria such as the Mogi-Coulomb is an advantage in wellbore stability analysis.

Simangunsong et al. (2006) compared minimum mud weight prediction of three common rock failure criteria. By using linear elasticity borehole stability model, the Mohr-Coulomb, the Drucker-Prager, and the Modified Lade criteria were evaluated based on rock mechanical and stress data from North Sea fields (McLean and Addis, 1990, Wong et al., 1994). The three failure criteria was used to estimate the minimum required mud weigh required to drill the reservoir section in Cyrus field and a North Sea oil field. The results were compared with the actual mud weight used to successfully drill the borehole. According to the results, the Circumscribed Drucker-Prager underestimated the minimum required mud weight to prevent breakouts from occurring, while the Mohr-Coulomb criterion overestimates the required minimum required mud weight to prevent borehole breakouts. The estimated results of minimum required mud weight by the Modified Lade criterion is not as conservative as the Mohr-Coulomb and not as unsafe as the Drucker-Prager and therefore, the Modified Lade criterion predicts more realistic results than the other two failure criteria.

Benz and Schwab (2008) optimized the material parameters for six rock failure criteria to obtain the least misfit with polyaxial test data of eight different types of rock. They concluded that non-linearity and including the effects of intermediate principal stress are two features of failure criteria which possibly enhance better fitting of polyaxial test data. Non-linear polyaxial HBMN criterion (Matsuoka and Nakai, 1982) which is an extension of the Hoek-Brown gave a good fit with polyaxial test data compared to other failure criteria.

Nawrocki (2010) predicted borehole breakout pressure based on the evaluation of four rock failure criteria including the Mohr-Coulomb, the Modified Lade, the Inscribed and Circumscribed Drucker-Prager. Laboratory measured rock mechanical properties for two different types of sandstone were used in his analysis. He recommended the Modified Lade as a reliable failure criterion for wellbore stability analysis. Vertical stress gradient determined from rock density and horizontal stresses were calculated according to typical range of effective horizontal to vertical stress ratio. Based on his results, the Inscribed Drucker-Prager and the Mohr-Coulomb predict higher minimum required mud weight compared to other two failure criteria since they underestimate the strength of rock. The Circumscribed Drucker-Prager estimates the lowest range of borehole breakout pressure by overestimation of rock strength. The Modified lade predicts the borehole breakout pressure in the middle range of results between upper and lower boundaries by other criteria. Furthermore, the Modified Lade has minimum difference of results from the vertical to horizontal borehole compared to other three failure criteria. Therefore, Nawrocki concluded that the Modified Lade criterion can be a reliable failure criterion for the practical purposes.

Zhang, Kao, and Radha (2010) compared minimum mud weight prediction of five common failure criteria including the Mohr-Coulomb, the Mogi-Coulomb, the Modified Lade, the Hoek-Brown and the Drucker-Prager. The results of triaxial tests were used to determine material parameters of failure criteria for different lithology. The hypothetical data set for in-situ stresses and also well depth (28000m for Dunham Dolomite and 12000m for Mizhou Trachyte) was considered. The Mohr-Coulomb and the Drucker-Prager criteria give upper and lower bound of results for the minimum required mud weight respectively. The Modified Lade criterion was considered as unsafe criterion for wellbore stability analysis due to overestimation of rock strength which might be significant. The Mogi-Coulomb and the Hoek-Brown criteria were recommended for wellbore stability analysis since the overestimation and underestimation of rock strength by these two failure criteria are small and can be neglected.

Reviewing previous studies reveals that some failure criteria, including Stassi d'Alia, have not been considered. Also in some of the previous studies, hypothetical data set have been used for the stress data, rock mechanical properties, and well depth which caused results to be unrealistic in some cases. For instance, true vertical well depth in the range of 12,000 m to 28,000 m has been chosen for analysis which caused the results not to be directly applicable to the stability of wells for petroleum exploitation (Zhang et al., 2010). Furthermore, quantitative comparisons have been previously studied on some of failure criteria but few evaluations of the failure criteria are based on typical petroleum related situations.

# 3. METHODOLOGY

Statistical analysis of rock failure criteria is the primary objective of this study. The thirteen of most common failure criteria have been described and the solution for the required material parameters was obtained based on the rock mechanical data from Rulison field in western Colorado (Higgings, 2006). A linear poro-elasticity model was chosen for determination of the borehole principal in-situ stresses. Variables for the statistical analysis can be divided in three groups including rock mechanical parameters, stress data, and well path parameters. Internal angle of friction and uniaxial compressive strength are the rock mechanical variables. Overburden, horizontal stresses and pore pressure are the effective stress variables for different depths. Different borehole inclination and azimuth was selected for the statistical analysis. Three different rock lithology; shale, sandstone, and siltstone were investigated. Similarities and differences of results for minimum required mud weight by different rock failure statistically investigated using percentage difference method and results presented through the table of contradiction. Different range of percentage difference has been specified by color code to highlight similarities and differences.

## 3.1. ROCK FAILURE CRITERIA

In the literature part of thesis (Section 2.2), selected failure criteria for this study comprehensively described and desired characteristics and disadvantages of each failure criterion has been discussed. Two main concerns which stand out for the evaluation of rock failure criteria are how to determine material constant parameters for failure criteria and also how to define numerical solution for determination of borehole breakout pressure with each different rock failure criterion.  This section will explain method has been used in this study to determine material parameters for different criteria.

**3.1.1. Failure Criteria Parameters.** One of the major challenges in studying rock failure criteria is the determination of material parameters for each criterion to have good fit with polyaxial test results. Previous studies have investigated mentioned problem and solution for determination of material parameters provided using the basic rock mechanical properties including uniaxial compressive strength (UCS) and frictional angle which are basic parameters of Mohr-Coulomb failure criterion (Vekeens and Walters, 1989, Zhou, 1994, Ewy, 1999, Al-Ajmi and Zimmerman, 2005). In this study, frictional angle and uniaxial compressive strength from triaxial tests results have been used as basic input data for the determination of material parameters for failure criteria. For some of failure criteria, flow factor ($q$) is one of the input for the solution of material parameters which is function of internal angle of friction. For those failure criteria with tensile strength component, uniaxial compressive strength data is used to estimate tensile strength. Hoek-Brown failure criterion is an exception since there is no published solution for the relation between material constant, $m$ and rock mechanical properties. Different rock mechanical references provided range of material constant, $m$ for different rock lithology (Jaeger et al., 2007, Zoback, 2007).

## 3.2. ROCK MECHANICAL & STRESS DATA

Rock mechanical and stress data from the Rulison field in western Colorado have been used in this study because the complete set of triaxial test results and stress data for different lithologies and effective stress state was reported (Higgins, 2006). The Rulison field is a tight gas play of interbedded sandstone, shale and siltstone. Rock mechanical data from triaxial tests on core samples of shale, sandstone and siltstone was reported. Stress data determined by a one dimensional geomechanical model (Figure 3.1 and 3.2). Based on the results of mechanical properties from triaxial tests, each rock lithology has been categorized into three groups of weak, medium and hard strength. One of the main reasons of using the data set from the Rulison field is its earth stress field. Horizontal compression of the Rocky Mountains causes the maximum horizontal stress gradient to be the largest stress component for the deeper sections. Stress data for well RWF 323-21 is shown in Table 3.2 where the well is in a normal faulting regime but for the pressurized zone at 2500 m depth, magnitude of maximum horizontal stress gradient is approaching the vertical stress gradient (Table 3.2 & Figure 3.1). Stress data for well RMV 60-17 shows a strike slip faulting regime (Table 3.3 & Figure 3.2).

Table 3.1. Rock mechanical properties

| Strength Level | Shale | | | Sandstone | | | Siltstone | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\phi$ (°) | UCS (MPa) | ν | $\phi$ (°) | UCS (MPa) | ν | $\phi$ (°) | UCS (MPa) | ν |
| Weak | 22 | 6 | 0.1 | 40 | 11 | 0.21 | 50 | 15 | 0.14 |
| Medium | 15 | 9 | 0.23 | 33 | 16 | 0.15 | 35 | 30 | 0.2 |
| Hard | 7 | 17 | 0.15 | 33 | 24 | 0.2 | 8 | 37 | 0.18 |

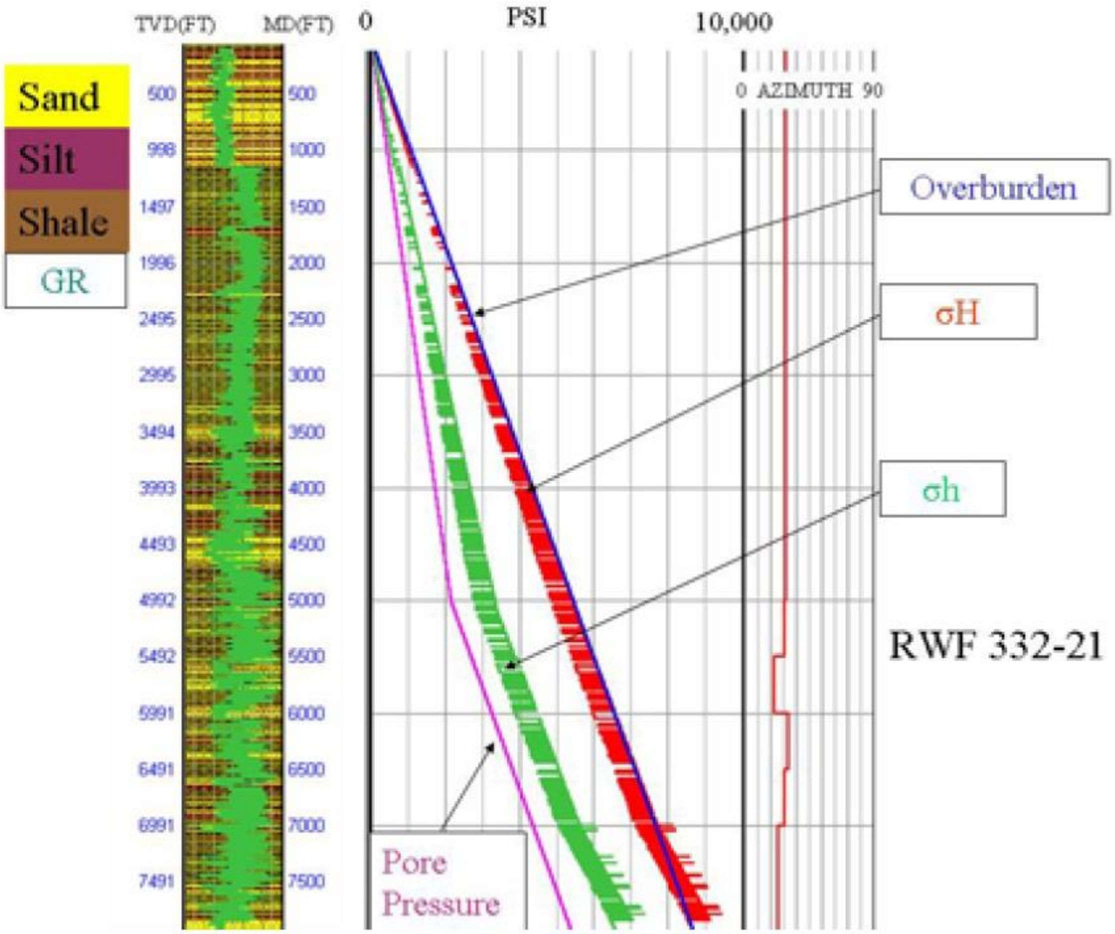


Figure 3.1. Stress Modeling Results for the Well RWF 332-21(Higgins, 2006)

Table 3.2. Stress data for well RWF 332-21, Normal Faulting regime

| Depth (m) | $\sigma_h$ (g/cc) | $\sigma_H$ (g/cc) | $\sigma_V$ (g/cc) | $p_0$ (g/cc) |
|---|---|---|---|---|
| 1500 | 1.51 | 2.27 | 2.52 | 1.02 |
| 2000 | 1.75 | 2.3 | 2.53 | 1.4 |
| 2500 | 2 | 2.5 | 2.5 | 1.6 |

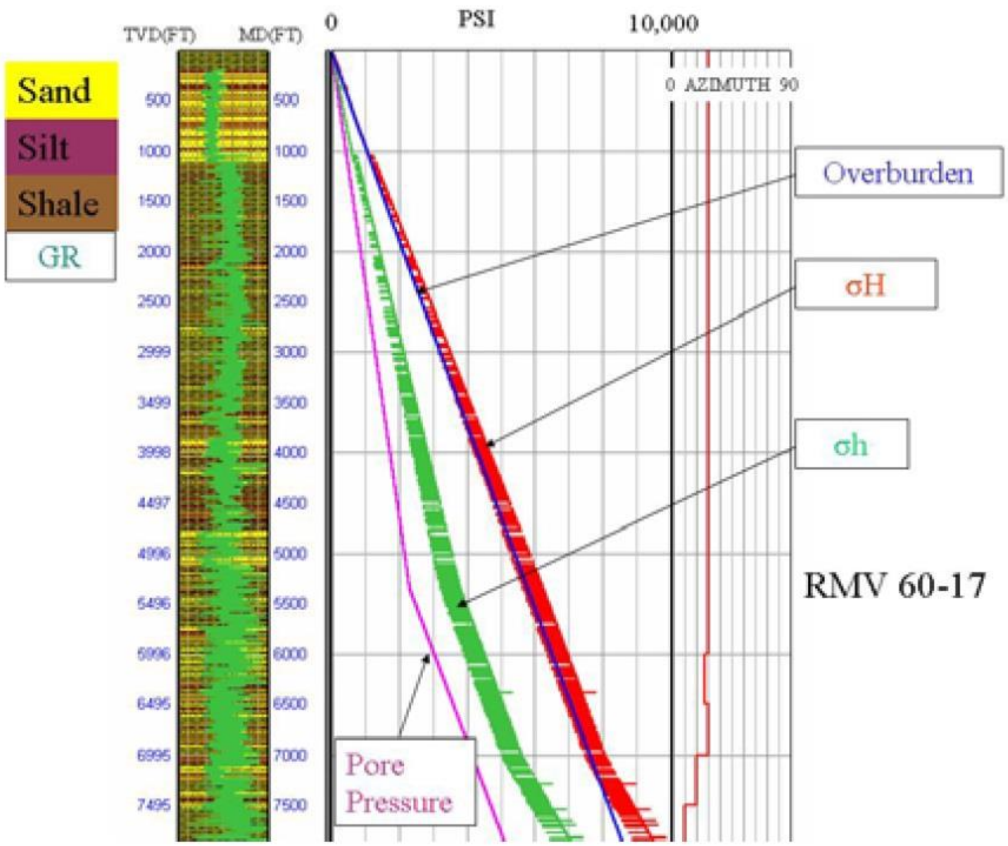


Figure 3.2. Stress Modeling Results for the well RMV 60-17 (Higgins, 2006)

Table 3.3. Stress data for Well RMV 60-17, Strike Slip faulting regime

| Depth (m) | $\sigma_h$ (g/cc) | $\sigma_H$ (g/cc) | $\sigma_V$ (g/cc) | $p_0$ (g/cc) |
|---|---|---|---|---|
| 1500 | 1.53 | 2.7 | 2.5 | 1.05 |
| 2000 | 1.73 | 2.65 | 2.5 | 1.3 |
| 2500 | 2.05 | 2.8 | 2.5 | 1.5 |

## 3.3. STATISTICAL ANALYSIS PROCEDURE

Based on the combination of the variables, different scenarios were defined for the statistical analysis. Rock properties of each different lithology (Table 3.1) were analyzed for the stress data in the Table 3.2 and Table 3.3 to determine the minimum drilling fluid density by rock failure criteria. All scenarios have been evaluated for a different well inclination and azimuth. Results of minimum required mud weight by different rock failure criteria for each specific scenario have been presented in a typical plot of minimum mud weight vs wellbore inclination. In order to statistically compare the results for different rock failure criteria, percentage difference method has been used and the results have been presented through the table of contradiction. The horizontal axis of contradiction table shows results of the minimum drilling fluid density for the horizontal borehole and the vertical axis represents the results for the vertical borehole. A color code has been specified for each interval of percentage difference. Percentage difference interval of 0 to 5 which highlighted with orange color shows the high level similarity in the results. The part of contradiction table which is highlighted by red color shows the high level of difference. Lower level of difference is shown by blue color and yellow color represents the lower range of similarity (Table 3.4).

Table 3.4. Color code for different interval of percentage difference (PD) in table of contradiction

| | |
|---|---|
| Orange | [-5 to 5] |
| Yellow | [-15 to -6] & [6 to 15] |
| Blue | [-30 to -16] & [16 to 30] |
| Red | [ PD <-30] & [30 <PD] |

# 4. RESULTS

The results of minimum required mud weight by different failure criteria are presented for three lithologies, shale, sandstone, and siltstone. The plot of minimum mud weight versus borehole inclination shows the results by different criteria for each scenario. Statistical comparison was done using percentage difference method and the results presented through the table of contradiction. Similarities and differences between the results of different rock failure criteria are investigated by interpretation of color mapping in tables of contradiction which represents the distribution of results. The effect of variation in rock mechanical properties on the results of different failure criteria for the minimum required mud weight was analyzed. The failure criteria on the boundaries of results, failure criteria with highest degree of similarity in results, and criterion with lowest difference between the results for the vertical and horizontal borehole are specified for different scenario. Also, the effect of strike-slip stress faulting regime on the distribution of results is interpreted. Furthermore, difference between the distribution of results for the normally pressurized zone and over pressurized zone is analyzed.

## 4.1. SHALE

For the weak shale which has the lowest UCS value in the series of rock mechanical data, the Tresca and the Von Mises give the upper high range of results and the Circumscribed Drucker-Prager predicts the lowest range of results. The Circumscribed Drucker-Prager approached the middle range of results by increasing inclination and show similar results as the Modified Lade for the horizontal borehole. The Mohr-Coulomb has minimum difference of results from vertical borehole to horizontal borehole and the Von Mises has maximum difference (Fig 4.1 and Table 4.1). The Modified Wiebols-Cook and the Mogi-Coulomb show closest match. For the wellbore case with 30 degree azimuth, there is difference in the distribution of results. In this condition, the Hoek-Brown and the Inscribed Drucker-Prager estimated the highest and lowest boundary of results for the vertical borehole. The Modified Wiebols-Cook and the Mogi-Coulomb predicted very similar results for a case with 30 degree azimuth. The Circumscribed Drucker-Prager predicts same results as the Modified Lade for the horizontal borehole (Table 4.2).

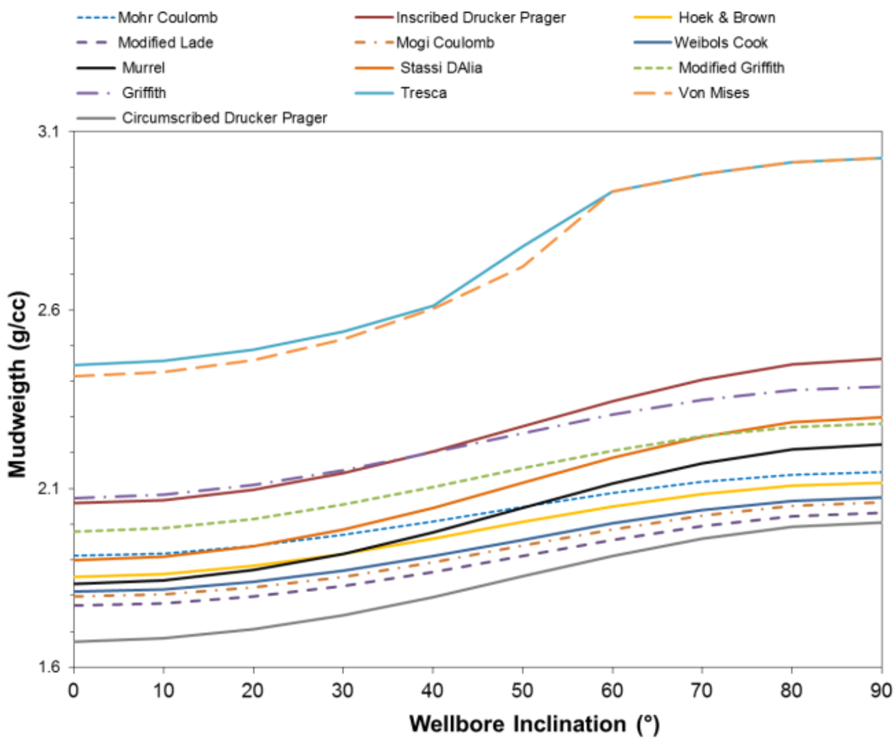


Figure 4.1. Minimum mud weight vs wellbore inclination by different failure criteria at 1500m depth and azimuth of 0° for weak shale

Table 4.1. Comparison of results by different failure criteria for weak shale at depth of 1500m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 1 | 3 | 4 | 11 | 6 | 15 | 7 | 14 | 23 | 19 | 51 | 51 |
| | ML | -6 | | 1 | 2 | 9 | 4 | 13 | 6 | 12 | 21 | 17 | 49 | 49 |
| | MG | -8 | -1 | | 1 | 8 | 3 | 12 | 4 | 11 | 19 | 15 | 47 | 47 |
| | MWC | -8 | -2 | -1 | | 7 | 2 | 11 | 3 | 10 | 19 | 14 | 46 | 46 |
| | MR | -10 | -3 | -2 | -1 | | -5 | 3 | -4 | 3 | 11 | 7 | 36 | 36 |
| | HB | -11 | -4 | -3 | -2 | -1 | | 9 | 1 | 8 | 16 | 12 | 43 | 43 |
| | SD | -14 | -7 | -6 | -5 | -4 | -2 | | -7 | -1 | 7 | 3 | 32 | 32 |
| | MC | -14 | -8 | -6 | -6 | -4 | -3 | -1 | | 6 | 15 | 11 | 41 | 41 |
| | MGR | -18 | -12 | -10 | -9 | -8 | -7 | -4 | -4 | | 8 | 4 | 33 | 33 |
| | IDP | -23 | -16 | -15 | -14 | -12 | -11 | -8 | -8 | -4 | | -4 | 23 | 23 |
| | GR | -24 | -17 | -15 | -14 | -13 | -12 | -9 | -8 | -5 | -1 | | 27 | 27 |
| | VM | -44 | -36 | -34 | -33 | -32 | -30 | -27 | -26 | -22 | -17 | -16 | | 0 |
| | TR | -46 | -38 | -36 | -35 | -33 | -32 | -29 | -28 | -24 | -19 | -18 | -1 | |

Table 4.2. Comparison of results by different failure criteria for weak shale at depth of 1500m and Azimuth of 30°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 0 | 1 | 2 | 9 | 5 | 14 | 4 | 10 | 23 | 19 | 46 | 54 |
| | ML | -2 | | 1 | 1 | 9 | 5 | 14 | 3 | 9 | 22 | 18 | 45 | 53 |
| | MG | -4 | -2 | | 0 | 8 | 4 | 13 | 2 | 8 | 21 | 17 | 44 | 51 |
| | MWC | -4 | -2 | -1 | | 8 | 3 | 12 | 2 | 8 | 21 | 17 | 43 | 51 |
| | MR | -11 | -9 | -7 | -7 | | -4 | 4 | -5 | 0 | 12 | 9 | 33 | 40 |
| | HB | 1 | 3 | 4 | 5 | 11 | | 9 | -1 | 4 | 17 | 13 | 38 | 46 |
| | SD | -17 | -14 | -13 | -12 | -5 | -17 | | -9 | -4 | 8 | 4 | 27 | 34 |
| | MC | -8 | -6 | -5 | -4 | 3 | -9 | 7 | | 6 | 19 | 15 | 40 | 48 |
| | MGR | -15 | -12 | -10 | -10 | -3 | -15 | 2 | -6 | | 12 | 8 | 33 | 40 |
| | IDP | -42 | -39 | -37 | -36 | -27 | -43 | -22 | -31 | -24 | | -3 | 18 | 25 |
| | GR | -29 | -27 | -24 | -24 | -16 | -30 | -11 | -19 | -13 | 9 | | 23 | 29 |
| | VM | -40 | -37 | -35 | -34 | -26 | -41 | -20 | -29 | -22 | 1 | -8 | | 5 |
| | TR | -41 | -38 | -36 | -35 | -26 | -41 | -20 | -30 | -23 | 1 | -9 | 0 | |

The failure criteria show different trends of results for horizontal wells oriented with a 90 degree from North. The differences between highest and lowest boundary of results for vertical borehole is higher than other cases (Tables 4.3 and 4.5). Distribution of results is less than other cases except the higher boundary of results for the vertical borehole. Failure criteria in the high range estimated the same results for the horizontal borehole (Tables 4.3 and 4.5). The Modified Wiebols-Cook and the Mogi-Coulomb predicted very similar results in all cases studied for the weak shale. Although, the Circumscribed Drucker-Prager predicted the lowest range of results but its difference with failure criteria in the middle range of results is relatively small especially for the horizontal borehole (Tables 4.1, 4.2 and 4.4). Distribution of results for the deeper depth in over pressurized zone is lower than normally pressurized zone in shallower depth (Tables 4.1, 4.4, and 4.7). For over pressurized zone at 2000 m depth, the Tresca and the Circumscribed Drucker-Prager predicted the higher and lower bound of results respectively (Figure 4.3 and Tables 4.4 and 4.5). In all cases studied for the weak shale, the results of the Tresca and the Von Mises as higher bound have major difference with the results of other failure criteria (Figures 4.1 and 4.2)

Table 4.3. Comparison of results by different failure criteria for weak shale at depth of 1500m and Azimuth of 90°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 7 | 12 | 12 | 6 | 12 | 7 | 23 | 22 | 23 | 23 | 22 | 23 |
| | ML | -6 | | 5 | 4 | -1 | 4 | -1 | 14 | 14 | 14 | 14 | 14 | 15 |
| | MG | -8 | -1 | | 0 | -5 | 0 | -5 | 9 | 9 | 9 | 9 | 9 | 10 |
| | MWC | -8 | -2 | -1 | | -5 | 0 | -5 | 9 | 9 | 9 | 9 | 9 | 10 |
| | MR | -10 | -3 | -2 | -1 | | 6 | 0 | 15 | 15 | 15 | 15 | 15 | 16 |
| | HB | -11 | -4 | -3 | -2 | -1 | | -5 | 9 | 9 | 9 | 9 | 9 | 10 |
| | SD | -14 | -7 | -6 | -5 | -4 | -2 | | 15 | 15 | 15 | 15 | 15 | 16 |
| | MC | -14 | -8 | -6 | -6 | -4 | -3 | -1 | | 0 | 0 | 0 | 0 | 1 |
| | MGR | -18 | -12 | -10 | -9 | -8 | -7 | -4 | -4 | | 0 | 0 | 0 | 1 |
| | IDP | -23 | -16 | -15 | -14 | -12 | -11 | -8 | -8 | -4 | | 0 | 0 | 1 |
| | GR | -24 | -17 | -15 | -14 | -13 | -12 | -9 | -8 | -5 | -1 | | 0 | 1 |
| | VM | -44 | -36 | -34 | -33 | -32 | -30 | -27 | -26 | -22 | -17 | -16 | | 1 |
| | TR | -71 | -61 | -59 | -58 | -56 | -54 | -50 | -49 | -44 | -39 | -38 | -18 | |

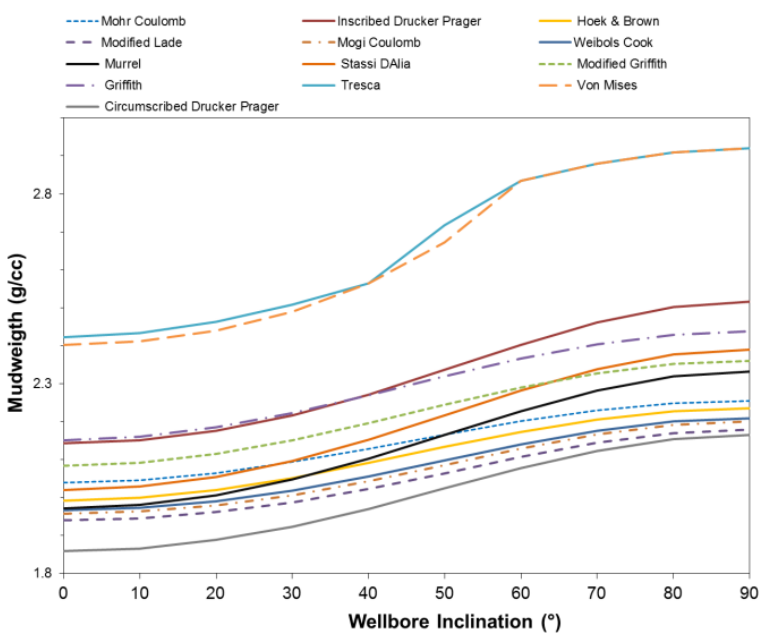


Figure 4.2. Minimum mud weight vs wellbore inclination by different failure criteria at 2000m depth and azimuth of 0° for weak shale

Table 4.4. Comparison of results by different failure criteria for weak shale at depth of 2000 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 1 | 2 | 2 | 8 | 3 | 10 | 4 | 9 | 16 | 13 | 35 | 35 |
| | ML | -4 | | 1 | 1 | 7 | 3 | 10 | 3 | 8 | 15 | 12 | 34 | 34 |
| | MG | -5 | -1 | | 0 | 6 | 2 | 9 | 2 | 7 | 14 | 11 | 33 | 33 |
| | MWC | -6 | -1 | 0 | | 5 | 1 | 8 | 2 | 7 | 14 | 10 | 32 | 32 |
| | MR | -6 | -2 | -1 | 0 | | -4 | 3 | -3 | 1 | 8 | 5 | 25 | 25 |
| | HB | -7 | -3 | -2 | -1 | -1 | | 7 | 1 | 6 | 13 | 9 | 31 | 31 |
| | SD | -9 | -4 | -3 | -3 | -3 | -1 | | -6 | -1 | 5 | 2 | 22 | 22 |
| | MC | -10 | -5 | -4 | -4 | -3 | -2 | -1 | | 5 | 12 | 8 | 30 | 30 |
| | MGR | -12 | -7 | -6 | -6 | -6 | -5 | -3 | -2 | | 7 | 3 | 24 | 24 |
| | IDP | -15 | -10 | -9 | -9 | -9 | -8 | -6 | -5 | -3 | | -3 | 16 | 16 |
| | GR | -16 | -11 | -10 | -9 | -9 | -8 | -7 | -6 | -3 | 0 | | 20 | 20 |
| | VM | -29 | -24 | -23 | -22 | -22 | -21 | -19 | -18 | -15 | -12 | -12 | | 0 |
| | TR | -30 | -25 | -24 | -23 | -23 | -22 | -20 | -19 | -16 | -13 | -13 | -1 | |

Table 4.5. Comparison of results by different failure criteria for weak shale at depth of 2000 m and Azimuth of 90°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 8 | 10 | 13 | 5 | 10 | 11 | 15 | 15 | 15 | 15 | 15 | 16 |
| | ML | -4 | | 2 | 5 | -2 | 2 | 3 | 7 | 7 | 7 | 7 | 7 | 8 |
| | MG | -5 | -1 | | 2 | -4 | 0 | 1 | 5 | 5 | 5 | 5 | 5 | 6 |
| | MWC | -6 | -1 | 0 | | -7 | -2 | -1 | 2 | 2 | 2 | 2 | 2 | 3 |
| | MR | -6 | -2 | -1 | 0 | | 5 | 6 | 10 | 10 | 10 | 10 | 10 | 11 |
| | HB | -7 | -3 | -2 | -1 | -1 | | 1 | 5 | 5 | 5 | 5 | 5 | 6 |
| | SD | -9 | -4 | -3 | -3 | -2 | -1 | | 4 | 4 | 4 | 4 | 4 | 5 |
| | MC | -10 | -5 | -4 | -4 | -3 | -2 | -1 | | 0 | 0 | 0 | 0 | 1 |
| | MGR | -12 | -7 | -6 | -6 | -6 | -5 | -3 | -2 | | 0 | 0 | 0 | 1 |
| | IDP | -15 | -10 | -9 | -9 | -9 | -8 | -6 | -5 | -3 | | 0 | 0 | 1 |
| | GR | -16 | -11 | -10 | -9 | -9 | -8 | -6 | -6 | -3 | 0 | | 0 | 1 |
| | VM | -29 | -24 | -23 | -22 | -22 | -21 | -19 | -18 | -15 | -12 | -12 | | 1 |
| | TR | -47 | -41 | -39 | -39 | -38 | -37 | -35 | -34 | -31 | -27 | -27 | -14 | |

When the rock mechanical properties of weak shale has been analyzed with the stress data of a strike slip faulting regime, the results show the same distribution as the normal faulting scenario but the difference is in a decreasing trend toward the horizontal borehole. The Mogi-Coulomb and the Modified Wiebols-Cook criteria show very close results in all cases have been studied for strike slip faulting regime in the weak shale (Figure 4.3, Tables 4.6 and 4.7). The Tresca and the Von Mises represented the highest bound of results with significant differences with the results of other failure criteria. The Mohr-Coulomb has the minimum difference between results for the vertical and horizontal borehole. The Circumscribed Drucker-Prager predicts same result as the Modified Lade for the vertical borehole (Figure 4.3). For a case with 90 degree azimuth, group of failure criteria estimated the same results for the horizontal borehole in the higher bound of results (Table 4.7). By increasing azimuth, distribution of results for the horizontal borehole tends to be lower (Table 4.7).

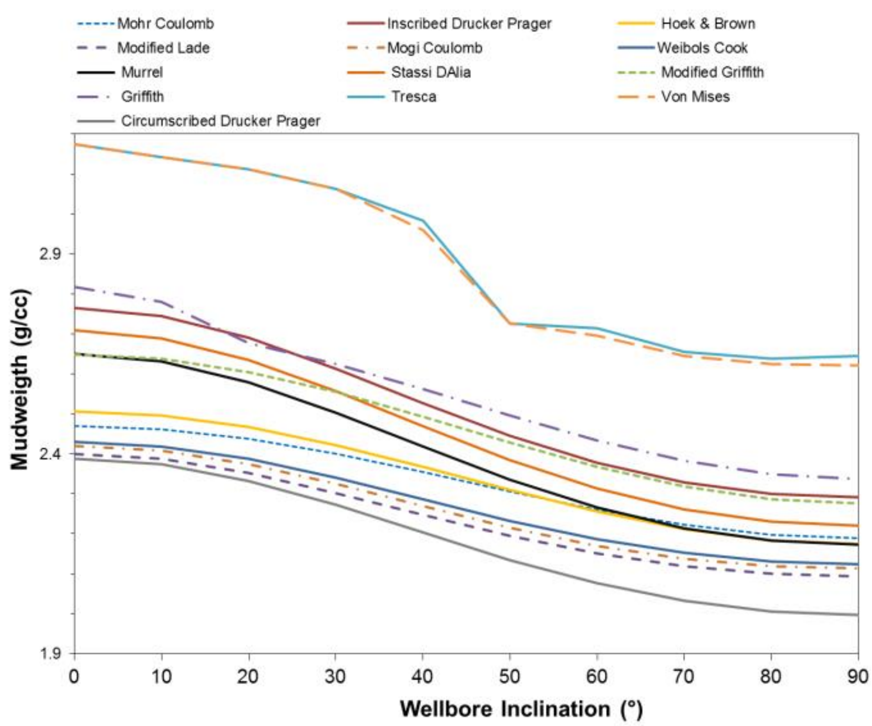


Figure 4.3. Minimum mud weight vs wellbore inclination by different failure criteria at 2500m depth and azimuth of 0° for weak shale under Strike Slip faulting regime

Table 4.6. Comparison of results by different failure criteria for weak shale at depth of 2500 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 5 | 6 | 6 | 9 | 9 | 11 | 10 | 14 | 15 | 17 | 31 | 32 |
| | ML | 0 | | 1 | 1 | 4 | 4 | 6 | 5 | 9 | 9 | 12 | 25 | 26 |
| | MG | -1 | -1 | | 0 | 3 | 3 | 5 | 4 | 8 | 8 | 11 | 24 | 25 |
| | MWC | -2 | -1 | 0 | | 2 | 2 | 5 | 3 | 7 | 8 | 10 | 23 | 25 |
| | MR | -11 | -10 | -10 | -9 | | 0 | 2 | 1 | 5 | 5 | 8 | 21 | 22 |
| | HB | -5 | -4 | -4 | -3 | 5 | | 2 | 1 | 5 | 5 | 8 | 21 | 22 |
| | SD | -13 | -13 | -12 | -11 | -2 | -8 | | -1 | 3 | 3 | 5 | 18 | 19 |
| | MC | -3 | -3 | -2 | -2 | 7 | 1 | 9 | | 4 | 5 | 7 | 20 | 21 |
| | MGR | -11 | -10 | -9 | -9 | 0 | -6 | 2 | -7 | | 0 | 3 | 15 | 16 |
| | IDP | -16 | -15 | -14 | -14 | -4 | -10 | -2 | -12 | -4 | | 2 | 14 | 15 |
| | GR | -18 | -17 | -16 | -16 | -6 | -12 | -4 | -14 | -6 | -2 | | 12 | 13 |
| | VM | -33 | -32 | -31 | -31 | -20 | -27 | -17 | -29 | -20 | -15 | -13 | | 1 |
| | TR | -33 | -32 | -31 | -31 | -20 | -27 | -17 | -29 | -20 | -15 | -13 | 0 | |

Table 4.7. Comparison of results by different failure criteria for weak shale at depth of 2500 m and Azimuth of 90°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 2 | 4 | 5 | 10 | 11 | 16 | 14 | 19 | 19 | 19 | 19 | 20 |
| | ML | 0 | | 2 | 3 | 8 | 8 | 14 | 11 | 17 | 17 | 17 | 17 | 17 |
| | MG | -1 | -1 | | 1 | 6 | 7 | 12 | 10 | 15 | 15 | 15 | 15 | 15 |
| | MWC | -2 | -1 | 0 | | 5 | 5 | 10 | 8 | 13 | 13 | 13 | 13 | 14 |
| | MR | -11 | -10 | -10 | -9 | | 1 | 5 | 3 | 8 | 8 | 8 | 8 | 9 |
| | HB | -5 | -4 | -4 | -3 | 5 | | 5 | 3 | 8 | 8 | 8 | 8 | 8 |
| | SD | -13 | -13 | -12 | -11 | -2 | -8 | | -2 | 3 | 3 | 3 | 3 | 3 |
| | MC | -3 | -3 | -2 | -2 | 7 | 1 | 9 | | 5 | 5 | 5 | 5 | 5 |
| | MGR | -11 | -10 | -9 | -9 | 0 | -6 | 2 | -7 | | 0 | 0 | 0 | 0 |
| | IDP | -16 | -15 | -14 | -14 | -4 | -10 | -2 | -12 | -4 | | 0 | 0 | 0 |
| | GR | -18 | -17 | -16 | -16 | -6 | -12 | -4 | -14 | -6 | -2 | | 0 | 0 |
| | VM | -36 | -35 | -34 | -34 | -22 | -29 | -20 | -31 | -22 | -17 | -15 | | 0 |
| | TR | -33 | -33 | -32 | -31 | -20 | -27 | -18 | -29 | -20 | -15 | -13 | 2 | |

For the medium strength and hard shale, higher UCS caused different failure criteria to be in the lower boundaries of results. The differences between failure criteria for the lower boundary and middle range of results are more significant in hard shale compare to medium strength shale. Greater magnitude of tensile component due to higher UCS is the main reason why the Murrel, the Stassi d'Alia, and the Modified Griffith criteria are in the lower boundary of results (Figures 4.4 and 4.5, Tables 4.8 and 4.9). The Tresca criterion represents the highest boundary of results and the Murrel predicts the lowest boundary of results. The Inscribed Drucker-Prager and the Modified Griffith replaced the Tresca and the Murrel respectively as the highest and lowest boundary of results by increasing inclination toward horizontal borehole and this is more dominant for the high azimuth cases (Table 4.9). In all cases studied for the hard shale, the Modified Lade, the Mogi-Coulomb, and the Modified Wiebols-Cook estimated very similar results (Figures 4.4 and 4.5, Tables 4.8 and 4.9). In this situation, the results of the Circumscribed Drucker-Prager is different than weak shale (Figure 4.1) and are very close to the results of the Modified Lade, the Mogi-Coulomb, and the Modified Wiebols-Cook criteria in the upper medium range of results (Figures 4.4 and 4.5, Tables 4.8 and 4.9).

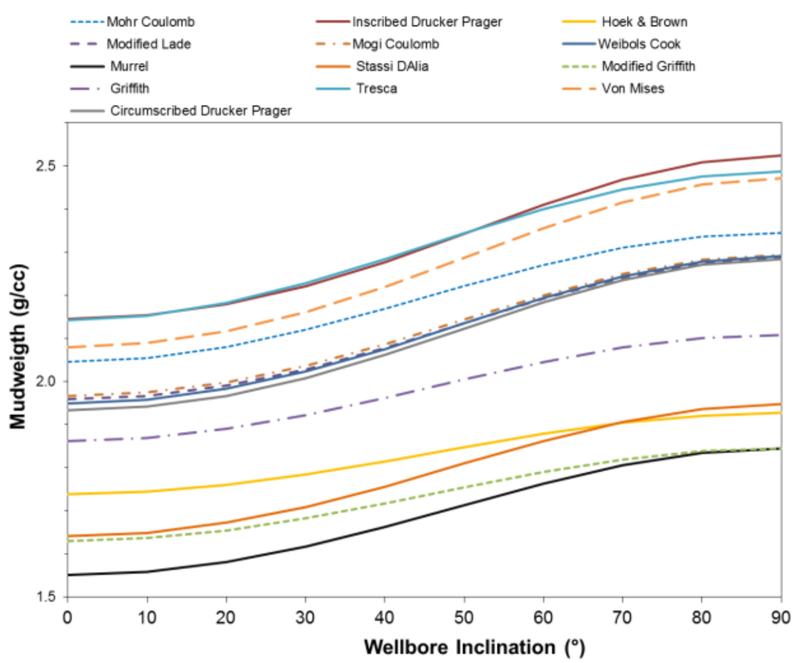


Figure 4.4. Minimum mud weight vs wellbore inclination by different failure criteria at 2000m depth and azimuth of 0° for hard shale

Table 4.8. Comparison of results by different failure criteria for the hard shale at depth 2000 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 0 | 0 | 0 | -19 | -16 | -15 | 3 | -19 | 11 | -8 | 8 | 9 |
| | ML | -1 | | 0 | 0 | -19 | -16 | -15 | 3 | -19 | 10 | -8 | 8 | 9 |
| | MG | -2 | 0 | | 0 | -20 | -16 | -15 | 2 | -20 | 10 | -8 | 8 | 8 |
| | MWC | -1 | 0 | 1 | | -19 | -16 | -15 | 2 | -19 | 10 | -8 | 8 | 9 |
| | MR | 20 | 21 | 21 | 20 | | 4 | 6 | 27 | 0 | 37 | 14 | 34 | 35 |
| | HB | 10 | 11 | 11 | 11 | -12 | | 1 | 22 | -4 | 31 | 9 | 28 | 29 |
| | SD | 15 | 16 | 17 | 16 | -6 | 6 | | 20 | -5 | 30 | 8 | 27 | 28 |
| | MC | -6 | -4 | -4 | -5 | -32 | -17 | -25 | | -21 | 8 | -10 | 5 | 6 |
| | MGR | 16 | 17 | 17 | 16 | -5 | 6 | 1 | 20 | | 37 | 14 | 34 | 35 |
| | IDP | -11 | -10 | -9 | -10 | -38 | -23 | -31 | -5 | -32 | | -16 | -2 | -1 |
| | GR | 4 | 5 | 5 | 5 | -20 | -7 | -13 | 9 | -14 | 13 | | 17 | 18 |
| | VM | -8 | -6 | -6 | -7 | -34 | -19 | -27 | -2 | -28 | 3 | -12 | | 1 |
| | TR | -11 | -9 | -9 | -10 | -38 | -23 | -31 | -5 | -31 | 0 | -15 | -3 | |

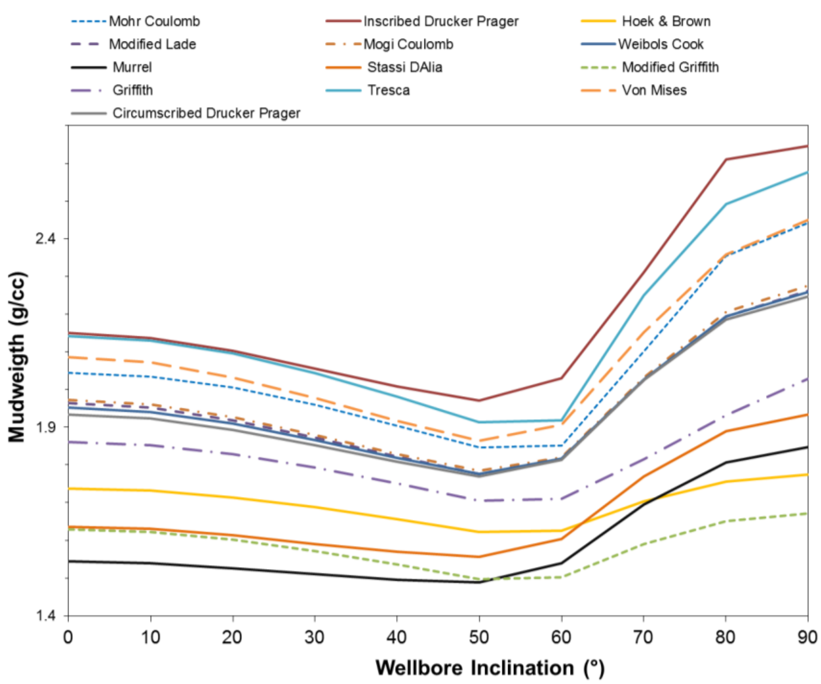


Figure 4.5. Minimum mud weight vs wellbore inclination by different failure criteria at 2000m depth and azimuth of 90° for hard shale

Table 4.9. Comparison of results by different failure criteria for the hard shale at depth of 2000 m and Azimuth of 90°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 1 | 2 | 1 | -18 | -22 | -14 | 9 | -26 | 17 | -8 | 10 | 15 |
| | ML | -1 | | 1 | 0 | -19 | -22 | -15 | 8 | -27 | 16 | -9 | 9 | 14 |
| | MG | -2 | 0 | | -1 | -19 | -23 | -15 | 8 | -27 | 15 | -10 | 8 | 13 |
| | MWC | -1 | 0 | 1 | | -18 | -22 | -15 | 9 | -27 | 16 | -9 | 9 | 14 |
| | MR | 20 | 21 | 21 | 20 | | -5 | 5 | 33 | -10 | 43 | 12 | 34 | 40 |
| | HB | 10 | 11 | 12 | 11 | -12 | | 10 | 40 | -6 | 49 | 17 | 40 | 47 |
| | SD | 15 | 16 | 17 | 16 | -6 | 5 | | 27 | -14 | 36 | 7 | 28 | 34 |
| | MC | -6 | -4 | -4 | -5 | -32 | -18 | -25 | | -33 | 7 | -16 | 1 | 5 |
| | MGR | 16 | 17 | 17 | 16 | -5 | 6 | 1 | 20 | | 59 | 24 | 49 | 56 |
| | IDP | -11 | -10 | -9 | -10 | -38 | -24 | -31 | -5 | -32 | | -22 | -6 | -2 |
| | GR | 4 | 5 | 5 | 5 | -20 | -7 | -13 | 9 | -14 | 13 | | 20 | 25 |
| | VM | -8 | -6 | -6 | -7 | -34 | -20 | -27 | -2 | -28 | 3 | -12 | | 4 |
| | TR | -11 | -9 | -9 | -10 | -38 | -24 | -31 | -5 | -31 | 0 | -15 | -3 | |

## 4.2. SANDSTONE

In the case of sandstone, the table of contradiction is dominated with red for the normally pressurized zone which shows a major difference in distribution of results (Figure 4.6, Tables 4.10). This is the same for the different level of strength in sandstone (Table 4.11). Higher internal angle of friction in the weak sandstone caused the results of the Circumscribed Drucker-Prager remarkably decrease and distribution of results be higher. This situation is different for the over pressurized zone (deeper depth) and distribution of results is lower than the normally pressurized zone (Tables 4.11). The Tresca and the Circumscribed Drucker-Prager criteria represent the highest and lowest boundaries of results respectively. The Modified Lade shows lowest difference of results from vertical to horizontal borehole (Figures 4.6). The Modified Lade and the Mogi-Coulomb criteria predicted very similar results in all cases for the weak sandstone. Failure criteria in the middle range of results show significant difference with the higher and lower boundaries of results (Figures 4.6 and 4.7).

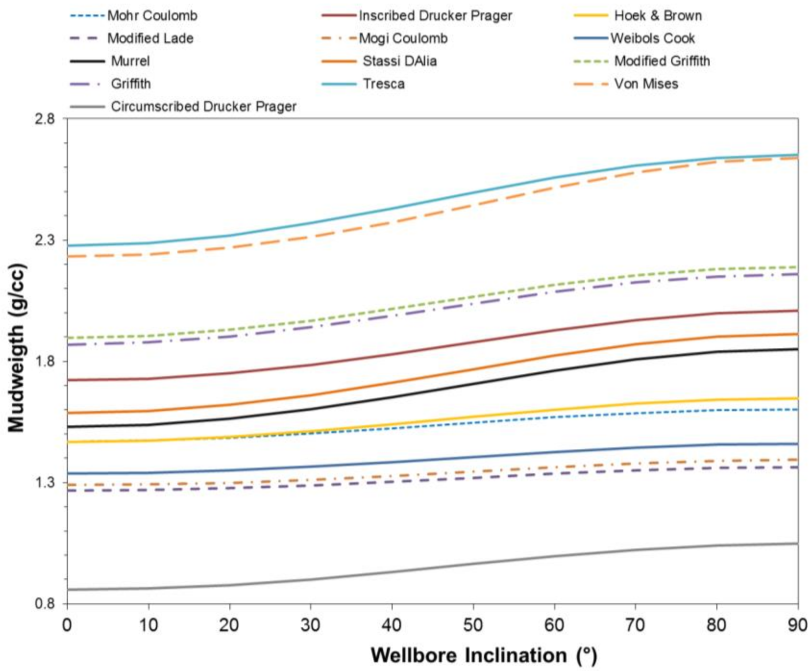


Figure 4.6. Minimum mud weight vs wellbore inclination by different failure criteria at 1500 m depth and azimuth of 0° for weak sandstone

Table 4.10. Comparison of results by different failure criteria for the weak sandstone at depth of 1500 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 30 | 33 | 39 | 77 | 57 | 83 | 53 | 106 | 92 | 106 | 152 | 153 |
| | ML | -48 | | 2 | 7 | 36 | 21 | 40 | 18 | 59 | 47 | 58 | 94 | 95 |
| | MG | -50 | -2 | | 5 | 33 | 18 | 37 | 15 | 55 | 44 | 55 | 89 | 90 |
| | MWC | -56 | -6 | -4 | | 27 | 13 | 31 | 10 | 48 | 38 | 48 | 81 | 82 |
| | MR | -78 | -21 | -19 | -14 | | -11 | 3 | -13 | 17 | 9 | 17 | 43 | 43 |
| | HB | -71 | -16 | -14 | -10 | 4 | | 16 | -3 | 31 | 22 | 31 | 60 | 61 |
| | SD | -85 | -25 | -23 | -19 | -4 | -8 | | -19 | 13 | 5 | 13 | 38 | 39 |
| | MC | -71 | -16 | -14 | -10 | 4 | 0 | 7 | | 35 | 25 | 35 | 65 | 65 |
| | MGR | -121 | -50 | -47 | -42 | -24 | -29 | -19 | -29 | | -7 | 1 | 22 | 23 |
| | IDP | -101 | -36 | -33 | -29 | -13 | -17 | -8 | -17 | 9 | | 8 | 31 | 32 |
| | GR | -118 | -48 | -45 | -40 | -22 | -27 | -18 | -27 | 1 | -9 | | 22 | 23 |
| | VM | -160 | -76 | -73 | -67 | -46 | -52 | -41 | -52 | -18 | -30 | -19 | | 1 |
| | TR | -165 | -80 | -76 | -70 | -49 | -55 | -43 | -55 | -20 | -32 | -22 | -2 | |

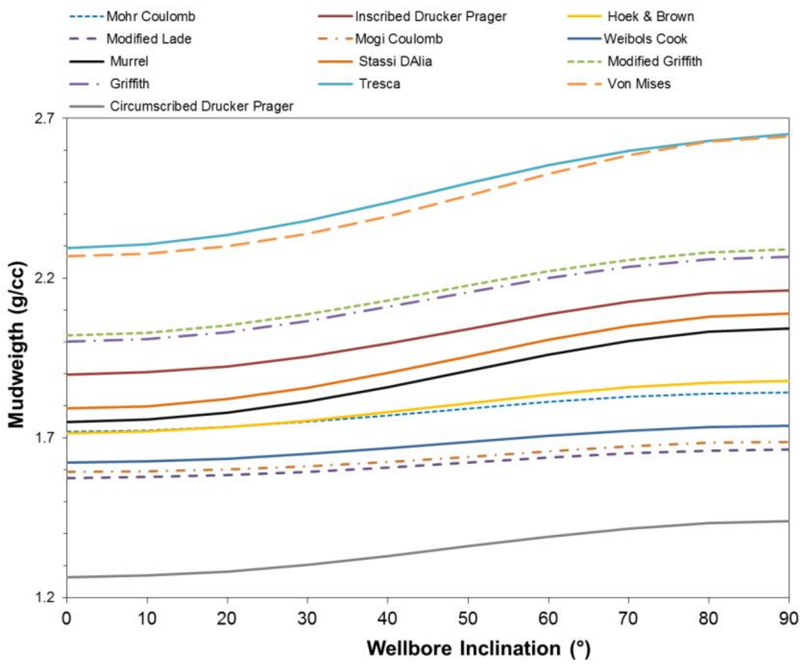


Figure 4.7. Minimum mud weight vs wellbore inclination by different failure criteria at 2000m depth and azimuth of 0° for weak sandstone

Table 4.11. Comparison of results by different failure criteria for the weak sandstone at depth of 2000 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 16 | 17 | 21 | 42 | 31 | 45 | 28 | 58 | 50 | 58 | 84 | 84 |
| | ML | -25 | | 1 | 4 | 23 | 13 | 26 | 11 | 37 | 30 | 36 | 59 | 59 |
| | MG | -26 | -1 | | 3 | 21 | 11 | 24 | 9 | 35 | 28 | 34 | 57 | 57 |
| | MWC | -28 | -3 | -2 | | 18 | 8 | 20 | 6 | 31 | 24 | 31 | 52 | 53 |
| | MR | -38 | -11 | -10 | -8 | | -8 | 2 | -10 | 11 | 6 | 11 | 29 | 30 |
| | HB | -36 | -9 | -8 | -6 | 2 | | 11 | -2 | 21 | 15 | 21 | 41 | 41 |
| | SD | -42 | -14 | -12 | -10 | -2 | -4 | | -12 | 9 | 3 | 8 | 26 | 27 |
| | MC | -36 | -9 | -8 | -6 | 2 | 0 | 4 | | 23 | 17 | 23 | 43 | 44 |
| | MGR | -60 | -28 | -27 | -24 | -16 | -18 | -13 | -18 | | -5 | 0 | 16 | 17 |
| | IDP | -50 | -21 | -19 | -17 | -9 | -11 | -6 | -10 | 6 | | 5 | 22 | 23 |
| | GR | -58 | -27 | -26 | -23 | -14 | -17 | -12 | -16 | 1 | -5 | | 17 | 17 |
| | VM | -80 | -44 | -42 | -40 | -30 | -32 | -27 | -32 | -12 | -20 | -13 | | 0 |
| | TR | -82 | -46 | -44 | -41 | -31 | -34 | -28 | -33 | -14 | -21 | -15 | -1 | |

Increasing UCS caused the Murrel and the Stassi d'Alia approach the middle range of results in medium strength sandstone and lower boundary of results for the vertical borehole in the hard sandstone (Figures 4.8, 4.9, and 4.10). The Tresca and the Circumscribed Drucker-Prager represent the highest and lowest boundaries of results respectively (Figures 4.8, 4.9, Tables 4.12). The results of the Inscribed Drucker-Prager moved toward the higher bound of results (Figures 4.8, 4.9, and 4.10). The Hoek-Brown replaced the Circumscribed Drucker-Prager as the lowest boundary of the results for the horizontal borehole by increasing the azimuth to 90 degree. In this situation, the Inscribed Drucker-Prager criterion estimated same result as the Tresca for the highest boundary of results (Figure 4.10 and Table 4.13). By increasing azimuth, the results of the Circumscribed Drucker-Prager approached the middle range of result with similarity to the results of the Modified Lade criterion for the horizontal borehole (Figure 4.10 and Table 4.13). The Hoek-Brown criterion has lowest difference of results from vertical to the horizontal borehole (Figures 4.8, 4.9, and 4.10, Tables 4.12 and 4.13). Distribution of the results is going to be lower by increasing wellbore azimuth (Tables 4.12 and 4.13).

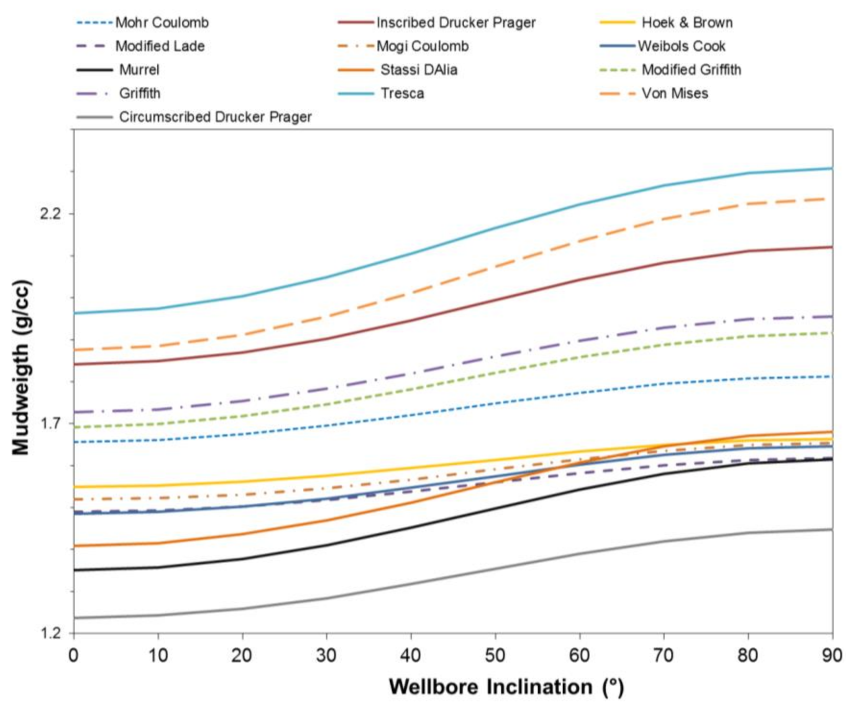


Figure 4.8. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 0° for hard sandstone

Table 4.12. Comparison of results by different failure criteria for hard sandstone at depth of 2000 m and Azimuth of 0°

| Vertical Borehole \ Horizontal Borehole | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CDP | | 12 | 14 | 14 | 12 | 15 | 16 | 25 | 32 | 47 | 35 | 55 | 59 |
| ML | -20 | | 2 | 2 | 0 | 3 | 4 | 12 | 18 | 31 | 21 | 38 | 43 |
| MG | -23 | -2 | | 0 | -2 | 1 | 2 | 10 | 16 | 28 | 18 | 35 | 40 |
| MWC | -20 | 0 | 2 | | -2 | 1 | 2 | 10 | 16 | 29 | 19 | 36 | 40 |
| MR | -9 | 9 | 11 | 9 | | 3 | 4 | 12 | 19 | 31 | 21 | 38 | 43 |
| HB | -25 | -4 | -2 | -4 | -15 | | 1 | 9 | 15 | 28 | 18 | 34 | 39 |
| SD | -14 | 5 | 7 | 5 | -4 | 9 | | 8 | 14 | 26 | 16 | 33 | 37 |
| MC | -34 | -11 | -9 | -12 | -23 | -7 | -18 | | 6 | 17 | 8 | 23 | 27 |
| MGR | -37 | -14 | -11 | -14 | -25 | -9 | -20 | -2 | | 11 | 2 | 17 | 20 |
| IDP | -49 | -24 | -21 | -24 | -36 | -19 | -31 | -11 | -9 | | -8 | 5 | 9 |
| GR | -40 | -16 | -14 | -16 | -28 | -12 | -23 | -4 | -2 | 6 | | 14 | 18 |
| VM | -52 | -26 | -24 | -26 | -39 | -21 | -33 | -13 | -11 | -2 | -9 | | 3 |
| TR | -59 | -32 | -29 | -32 | -45 | -27 | -39 | -19 | -16 | -7 | -14 | -5 | |

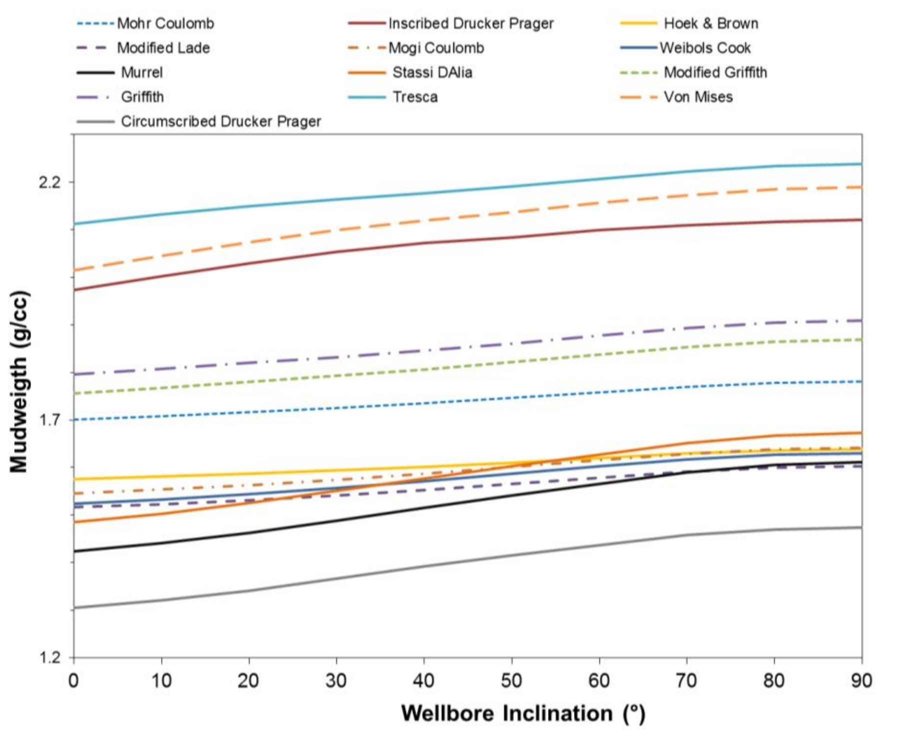


Figure 4.9. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 30° for hard sandstone

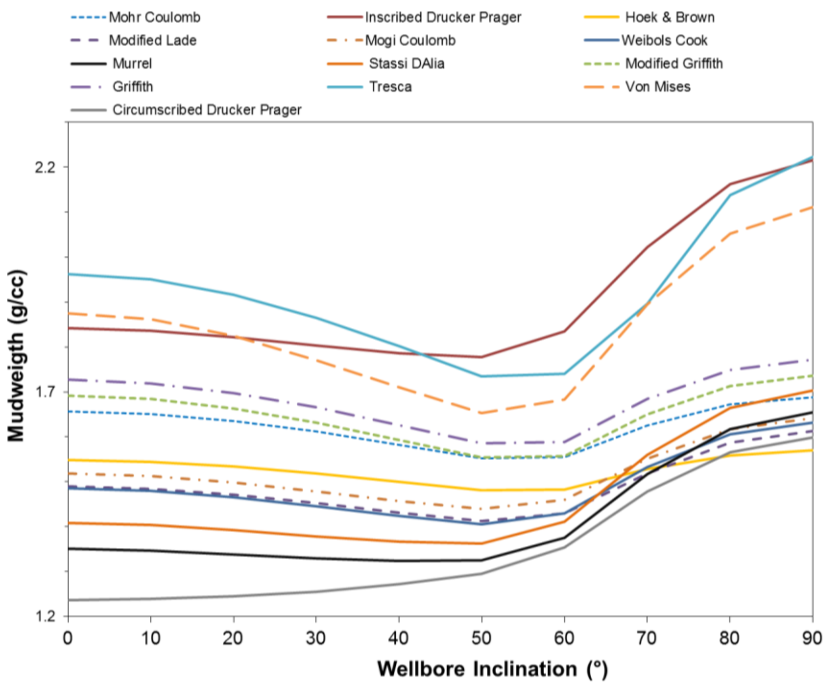


Figure 4.10. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 90° for hard sandstone

Table 4.13. Comparison of results by different failure criteria for the hard sandstone at depth of 2000 m and Azimuth of 90°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 1 | 3 | 2 | 4 | -2 | 7 | 6 | 9 | 39 | 11 | 32 | 39 |
| | ML | -20 | | 2 | 1 | 3 | -3 | 6 | 5 | 8 | 37 | 10 | 31 | 38 |
| | MG | -23 | -2 | | -1 | 1 | -4 | 4 | 3 | 6 | 35 | 8 | 29 | 35 |
| | MWC | -20 | 0 | 2 | | 1 | -4 | 4 | 3 | 6 | 36 | 9 | 29 | 36 |
| | MR | -9 | 9 | 11 | 9 | | -5 | 3 | 2 | 5 | 34 | 7 | 28 | 34 |
| | HB | -25 | -4 | -2 | -4 | -15 | | 8 | 7 | 11 | 41 | 13 | 34 | 42 |
| | SD | -14 | 5 | 7 | 5 | -4 | 9 | | -1 | 2 | 30 | 4 | 24 | 31 |
| | MC | -34 | -11 | -9 | -12 | -23 | -7 | -18 | | 3 | 31 | 5 | 25 | 32 |
| | MGR | -37 | -14 | -11 | -14 | -25 | -9 | -20 | -2 | | 28 | 2 | 22 | 28 |
| | IDP | -49 | -24 | -21 | -24 | -36 | -19 | -31 | -11 | -9 | | -20 | -5 | 0 |
| | GR | -40 | -16 | -14 | -16 | -28 | -12 | -23 | -4 | -2 | 6 | | 19 | 25 |
| | VM | -52 | -26 | -24 | -26 | -39 | -21 | -33 | -13 | -11 | -2 | -9 | | 5 |
| | TR | -59 | -32 | -29 | -32 | -45 | -27 | -39 | -19 | -16 | -7 | -14 | -5 | |

## 4.3. SILTSTONE

The weak siltstone has an average UCS but had the highest internal angle of friction between all cases of rock mechanical properties. In this condition, the Circumscribed Drucker-Prager as the lowest boundary of results has significant differences with the results of other failure criteria especially in normally pressurized zone (Figure 4.11 and Table 4.14). Higher frictional angle in the weak siltstone is the main reason for this distribution. The results of the Mogi-Coulomb and the Modified Lade criteria show close similarity in all cases studied weak siltstone (Figures 4.11 & 4.12, Tables 4.14 & 4.15). The results of the Tresca and the Circumscribed Drucker-Prager criteria show the highest difference. Difference between results of failure criteria for the vertical and horizontal borehole is smaller than other cases. For this situation, the Modified Lade criterion represents the lowest difference between results for the vertical and horizontal borehole (Figures 4.11 & 4.12). The failure criteria analysis for the weak siltstone for a normally pressurized zone show largest distribution between all different statistical analysis cases (Tables 4.14).

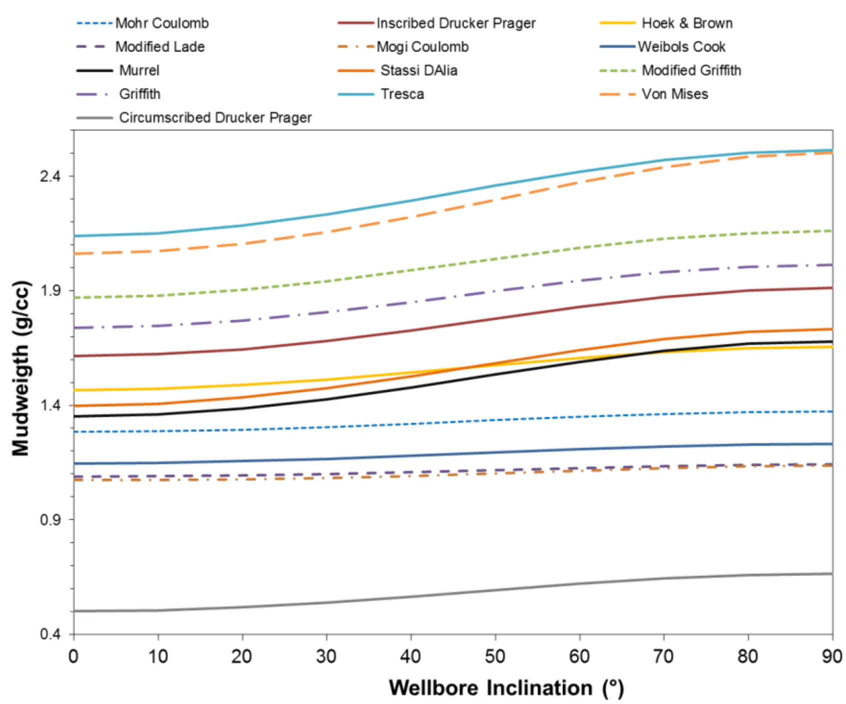


Figure 4.11. Minimum mud weight vs wellbore inclination by different failure criteria at 1500 m depth and azimuth of 0° for weak siltstone

Table 4.14. Comparison of results by different failure criteria for the weak siltstone at depth of 1500 m and Azimuth of 0°

| | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Vertical Borehole | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| CDP | | 72 | 71 | 85 | 152 | 149 | 160 | 106 | 225 | 188 | 203 | 276 | 278 |
| ML | -118 | | -1 | 8 | 47 | 45 | 52 | 20 | 89 | 68 | 76 | 119 | 120 |
| MG | -115 | 1 | | 8 | 48 | 46 | 53 | 21 | 90 | 68 | 77 | 120 | 121 |
| MWC | -129 | -5 | -7 | | 37 | 35 | 41 | 12 | 76 | 56 | 64 | 103 | 104 |
| MR | -170 | -24 | -26 | -18 | | -1 | 3 | -18 | 29 | 14 | 20 | 49 | 50 |
| HB | -193 | -35 | -37 | -28 | -9 | | 5 | -17 | 31 | 16 | 22 | 51 | 52 |
| SD | -180 | -28 | -30 | -22 | -4 | 5 | | -21 | 25 | 10 | 16 | 44 | 45 |
| MC | -157 | -18 | -20 | -12 | 5 | 12 | 8 | | 58 | 39 | 47 | 82 | 83 |
| MGR | -274 | -72 | -74 | -63 | -39 | -28 | -34 | -46 | | -11 | -7 | 16 | 16 |
| IDP | -223 | -48 | -51 | -41 | -20 | -10 | -15 | -26 | 14 | | 5 | 31 | 31 |
| GR | -248 | -60 | -62 | -52 | -29 | -18 | -24 | -35 | 7 | -8 | | 24 | 25 |
| VM | -312 | -90 | -92 | -80 | -53 | -41 | -48 | -61 | -10 | -28 | -19 | | 0 |
| TR | -328 | -97 | -99 | -87 | -58 | -46 | -53 | -67 | -14 | -33 | -23 | -4 | |

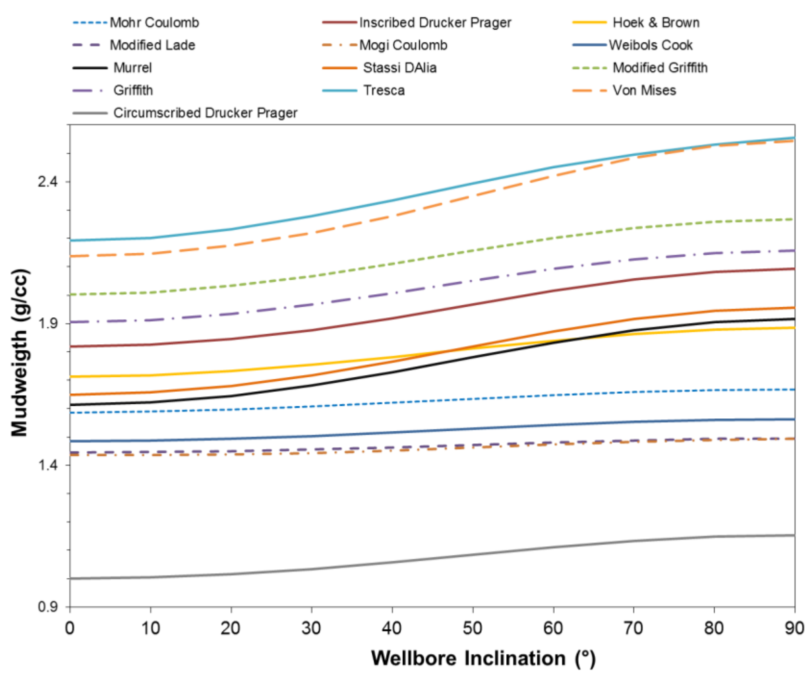


Figure 4.12. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 0° for weak siltstone

Table 4.15. Comparison of results by different failure criteria for the weak siltstone at depth of 2000 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 30 | 29 | 36 | 66 | 63 | 70 | 45 | 97 | 82 | 87 | 121 | 122 |
| | ML | -44 | | 0 | 5 | 28 | 26 | 31 | 12 | 52 | 40 | 44 | 70 | 71 |
| | MG | -43 | 1 | | 5 | 28 | 26 | 31 | 12 | 52 | 40 | 45 | 70 | 71 |
| | MWC | -48 | -3 | -4 | | 23 | 21 | 25 | 7 | 45 | 34 | 38 | 63 | 64 |
| | MR | -61 | -12 | -12 | -9 | | -2 | 2 | -13 | 18 | 9 | 13 | 33 | 33 |
| | HB | -71 | -18 | -19 | -15 | -6 | | 4 | -11 | 20 | 11 | 14 | 35 | 36 |
| | SD | -65 | -14 | -15 | -11 | -2 | 4 | | -15 | 16 | 7 | 10 | 30 | 31 |
| | MC | -58 | -10 | -11 | -7 | 2 | 7 | 4 | | 36 | 26 | 29 | 53 | 53 |
| | MGR | -100 | -38 | -39 | -35 | -24 | -17 | -21 | -26 | | -8 | -5 | 12 | 13 |
| | IDP | -82 | -26 | -27 | -22 | -13 | -6 | -10 | -15 | 9 | | 3 | 22 | 22 |
| | GR | -90 | -32 | -33 | -28 | -18 | -11 | -15 | -20 | 5 | -5 | | 18 | 18 |
| | VM | -114 | -48 | -49 | -44 | -32 | -25 | -30 | -35 | -7 | -18 | -12 | | 0 |
| | TR | -119 | -52 | -53 | -48 | -36 | -28 | -33 | -38 | -10 | -21 | -15 | -3 | |

The hard siltstone has the highest UCS and lowest frictional angle among all cases of rock mechanical properties. The Inscribed Drucker-Prager and the Murrel represented the highest and lowest boundaries of results respectively (Figure 4.13 and Table 4.16). By increasing azimuth, the Modified Griffith replaced the Murrel as the lowest boundary of the results for the horizontal borehole (Figures 4.14 & 4.15, Tables 4.17 & 4.18). The results of the Circumscribed Drucker-Prager criterion represented the middle range of results with close similarity to the results of the Modified Wiebols-Cook. The Modified Wiebols-Cook, the Circumscribed Drucker-Prager, the Mogi-Coulomb and the Modified Lade give very similar results for all cases studied for the hard siltstone. The Hoek-Brown criterion represented the lowest difference between results for the vertical and horizontal borehole (Figures 4.13, 4.14 and 4.15). Distribution of results for the horizontal borehole is lower for those cases with higher azimuth (Tables 4.17 & 4.18). Although the Von Mises usually predicted the higher bound of results for the minimum required mud weight, its results are in the middle range for the hard siltstone.

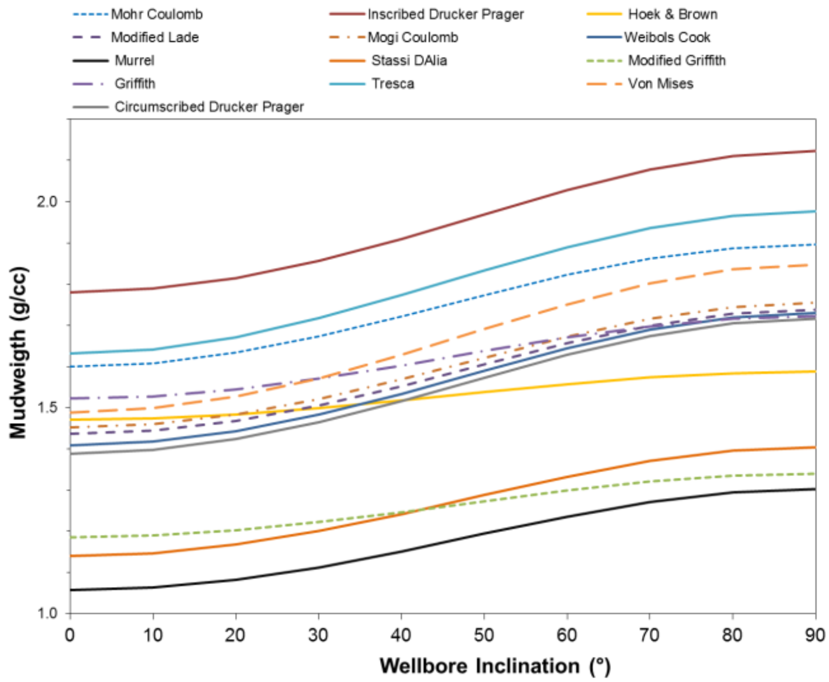


Figure 4.13. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 0° for hard siltstone

Table 4.16. Comparison of results by different failure criteria for the hard siltstone at depth 2000 m and Azimuth of 0°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 1 | 2 | 1 | -24 | -7 | -18 | 11 | -22 | 24 | 0 | 8 | 15 |
| | ML | -3 | | 1 | 0 | -25 | -9 | -19 | 9 | -23 | 22 | -1 | 6 | 14 |
| | MG | -5 | -1 | | -1 | -26 | -10 | -20 | 8 | -24 | 21 | -2 | 5 | 13 |
| | MWC | -1 | 2 | 3 | | -25 | -8 | -19 | 10 | -23 | 23 | 0 | 7 | 14 |
| | MR | 24 | 26 | 27 | 25 | | 22 | 8 | 46 | 3 | 63 | 32 | 42 | 52 |
| | HB | -6 | -2 | -1 | -4 | -39 | | -12 | 19 | -16 | 34 | 8 | 16 | 24 |
| | SD | 18 | 21 | 22 | 19 | -8 | 23 | | 35 | -5 | 51 | 23 | 32 | 41 |
| | MC | -15 | -11 | -10 | -14 | -51 | -9 | -40 | | -29 | 12 | -9 | -3 | 4 |
| | MGR | 15 | 17 | 18 | 16 | -12 | 19 | -4 | 26 | | 58 | 29 | 38 | 47 |
| | IDP | -28 | -24 | -23 | -26 | -69 | -21 | -56 | -11 | -50 | | -19 | -13 | -7 |
| | GR | -10 | -6 | -5 | -8 | -44 | -4 | -34 | 5 | -28 | 14 | | 7 | 15 |
| | VM | -7 | -4 | -3 | -6 | -41 | -1 | -31 | 7 | -26 | 16 | 2 | | 7 |
| | TR | -17 | -14 | -12 | -16 | -54 | -11 | -43 | -2 | -38 | 8 | -7 | -10 | |

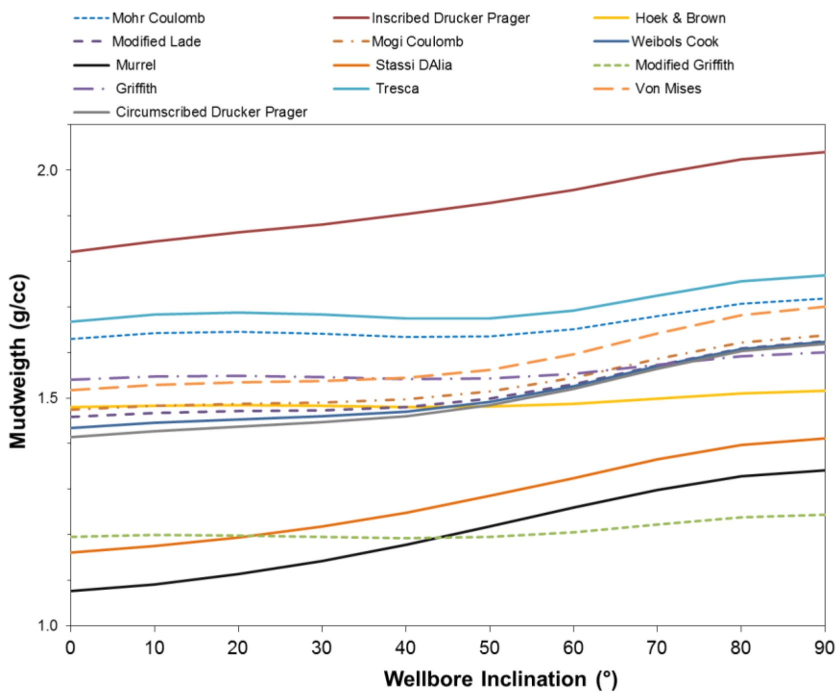


Figure 4.14. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 60° for hard siltstone

Table 4.17. Comparison of results by different failure criteria for the hard siltstone at depth 2000 m and Azimuth of 60°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 0 | 1 | 0 | -17 | -6 | -13 | 6 | 6 | 26 | -1 | 5 | 9 |
| | ML | -3 | | 1 | 0 | -17 | -7 | -13 | 6 | 6 | 26 | -2 | 5 | 9 |
| | MG | -4 | -1 | | -1 | -18 | -7 | -14 | 5 | 5 | 25 | -2 | 4 | 8 |
| | MWC | -1 | 2 | 3 | | -17 | -7 | -13 | 6 | 6 | 26 | -1 | 5 | 9 |
| | MR | 24 | 26 | 27 | 25 | | 13 | 5 | 28 | 28 | 52 | 19 | 27 | 32 |
| | HB | -5 | -1 | 0 | -3 | -37 | | -7 | 13 | 13 | 35 | 6 | 12 | 17 |
| | SD | 18 | 20 | 21 | 19 | -8 | 22 | | 22 | 22 | 45 | 13 | 20 | 25 |
| | MC | -15 | -12 | -11 | -14 | -51 | -10 | -40 | | 0 | 19 | -7 | -1 | 3 |
| | MGR | 16 | 18 | 19 | 17 | -11 | 19 | -3 | 27 | | 19 | -7 | -1 | 3 |
| | IDP | -29 | -25 | -24 | -27 | -69 | -23 | -57 | -12 | -52 | | -22 | -17 | -13 |
| | GR | -9 | -6 | -4 | -7 | -43 | -4 | -33 | 6 | -29 | 15 | | 6 | 11 |
| | VM | -7 | -4 | -3 | -6 | -41 | -3 | -31 | 7 | -27 | 17 | 1 | | 4 |
| | TR | -18 | -14 | -13 | -16 | -55 | -13 | -44 | -2 | -40 | 8 | -8 | -10 | |

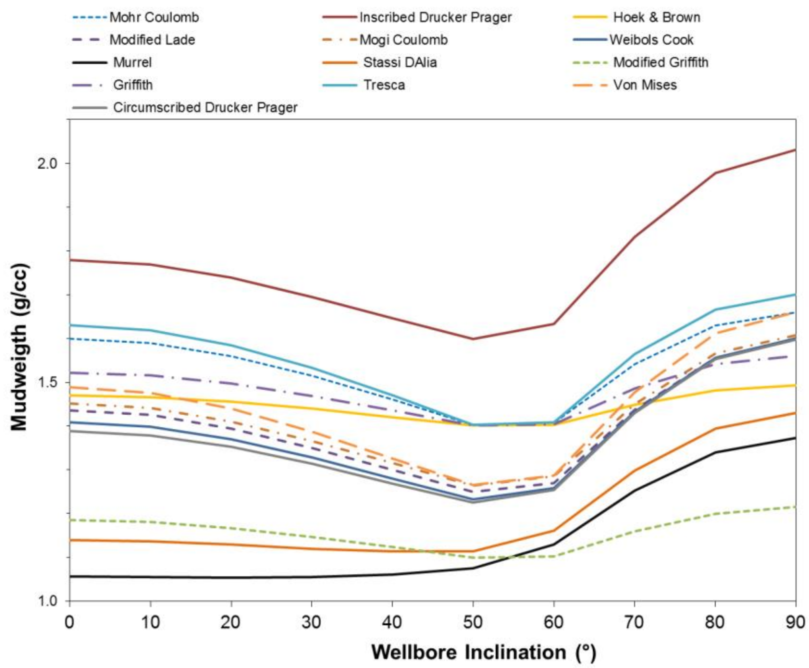


Figure 4.15. Minimum mud weight vs wellbore inclination by different failure criteria at 2000 m depth and azimuth of 90° for hard siltstone

Table 4.18. Comparison of results by different failure criteria for the hard siltstone at depth of 2000 m and Azimuth of 90°

| | | Horizontal Borehole | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CDP | ML | MG | MWC | MR | HB | SD | MC | MGR | IDP | GR | VM | TR |
| Vertical Borehole | CDP | | 0 | 1 | 0 | -14 | -7 | -12 | 4 | -24 | 27 | -2 | 4 | 6 |
| | ML | -3 | | 1 | 0 | -14 | -7 | -12 | 4 | -24 | 27 | -2 | 4 | 6 |
| | MG | -5 | -1 | | -1 | -15 | -7 | -12 | 3 | -25 | 26 | -3 | 3 | 6 |
| | MWC | -1 | 2 | 3 | | -14 | -7 | -12 | 4 | -24 | 27 | -2 | 4 | 6 |
| | MR | 24 | 26 | 27 | 25 | | 9 | 3 | 21 | -12 | 48 | 14 | 21 | 24 |
| | HB | -6 | -2 | -1 | -4 | -39 | | -5 | 11 | -19 | 36 | 5 | 11 | 14 |
| | SD | 18 | 21 | 22 | 19 | -8 | 23 | | 18 | -14 | 44 | 11 | 18 | 21 |
| | MC | -15 | -11 | -10 | -14 | -51 | -9 | -40 | | -27 | 22 | -6 | 0 | 3 |
| | MGR | 15 | 17 | 18 | 16 | -12 | 19 | -4 | 26 | | 68 | 29 | 37 | 41 |
| | IDP | -28 | -24 | -23 | -26 | -69 | -21 | -56 | -11 | -50 | | -23 | -18 | -16 |
| | GR | -10 | -6 | -5 | -8 | -44 | -4 | -34 | 5 | -28 | 14 | | 6 | 9 |
| | VM | -7 | -4 | -3 | -6 | -41 | -1 | -31 | 7 | -26 | 16 | 2 | | 2 |
| | TR | -17 | -14 | -12 | -16 | -54 | -11 | -43 | -2 | -38 | 8 | -7 | -10 | |

# 5. FIELD CASE STUDIES

Failure criteria were evaluated on three field cases in order to compare the predicted results of minimum required mud weight by field mud weight. A North Sea oil field (Wong et al., 1994), UK continental shelf (Mclean and Addis, 1990), and Pagerungan gas field (Ramos et al., 1998) are three fields which have been chosen for the evaluation of failure criteria since the complete set of rock mechanical properties, stress data and also information about drilling history (well trajectory and field mud weight) were provided. The percentage difference method was used to compare the results of minimum required mud weight by different failure criteria and field mud weight. The result of comparison is presented using the conventional column chart.

## 5.1. NORTH SEA OIL FIELD

Field data from the first extended-reach horizontal well drilled in North Sea oil field was used for the evaluation of failure criteria. Rock mechanical properties were determined based on the triaxial and thick walled cylinder strength (TWC) test results on core samples from three intervals, the Middle Ness (Sandstone with shale streaks), Etive (Sandstone), Rannoch (Sandstone) (Table 5.1). The deviated well plan is shown on figure 5.1. The extended-reach horizontal well was drilled successfully using the mud weight of 1.3 g/cc (0.57 psi/ft) for all three formations, Middle Ness, Etive and Rannoch.

Table 5.1. Field Data, North Sea Oil Field (Wong et al., 1994)

| Formation | Rock Properties | | | Stress Data | | | | Field MW (g/cc) |
|---|---|---|---|---|---|---|---|---|
| | $\phi$ (°) | $c$ (MPa) | $\nu$ | $\sigma_h$ (g/cc) | $\sigma_H$ (g/cc) | $\sigma_V$ (g/cc) | $p_0$ (g/cc) | |
| Middle Ness | 39 | 9 | 0.2 | 1.66 | 1.66 | 2.3 | 1.24 | 1.3 |
| Etive | 30 | 7 | 0.1 | 1.66 | 1.66 | 2.3 | 1.24 | 1.3 |
| Rannoch | 36 | 25 | 0.25 | 1.66 | 1.66 | 2.3 | 1.24 | 1.3 |

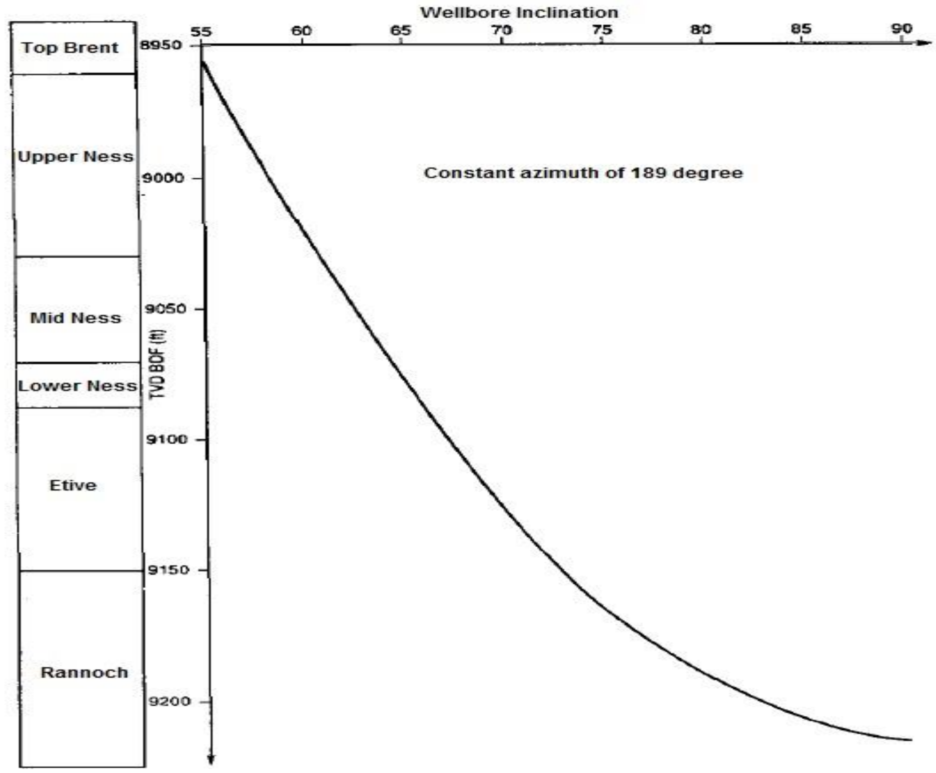


Figure 5.1. Deviated well plan, North Sea Oil Field (Wong et al., 1994)

Table 5.2. Results of predicted minimum required mud weight by different criteria and difference with field mud weight for Middle Ness formation (60° Inclination)

| Failure Criteria | Predicted Required Minimum Mud Weight (g/cc) | Percentage Difference with Field Mud Weight (%) |
|---|---|---|
| Ins. Drucker-Prager | 1.72 | 31.3 |
| Tresca | 1.67 | 27.6 |
| Von Mises | 1.59 | 21.4 |
| Modified Griffith | 1.50 | 14.7 |
| Griffith | 1.48 | 13.1 |
| Mohr-Coulomb | 1.40 | 6.8 |
| Hoek-Brown | 1.35 | 2.7 |
| Mogi-Coulomb | 1.27 | -3.2 |
| Modified Lade | 1.24 | -5.3 |
| Modified Wiebols-Cook | 1.24 | -5.5 |
| Stassi D'Alia | 1.24 | -5.5 |
| Murrel | 1.20 | -8.5 |
| Cir. Drucker-Prager | 1.08 | -17.7 |

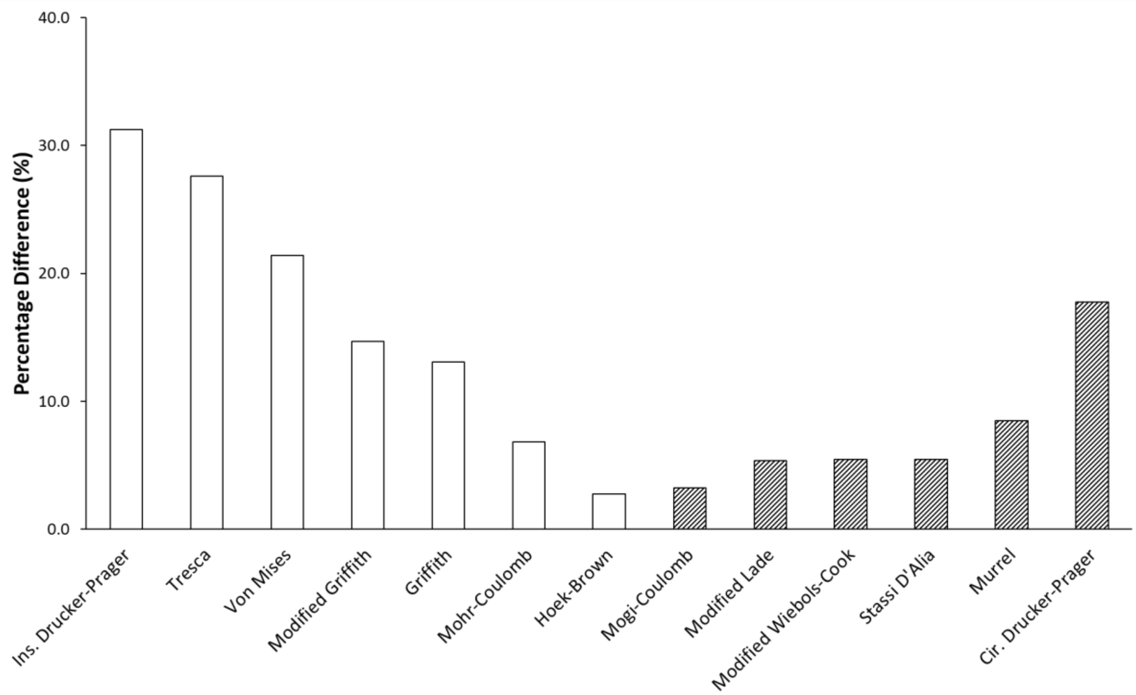


Figure 5.2. Comparison between results of predicted minimum required mud weight by different criteria and field mud weight for Middle Ness formation (60° Inclination)[1]

Table 5.3. Results of predicted minimum required mud weight by different criteria and difference with field mud weight for Etive formation (70° Inclination)

| Failure Criteria | Predicted Required Minimum Mud Weight (g/cc) | Percentage Difference with Field Mud Weight (%) |
|---|---|---|
| Tresca | 2.27 | 73.4 |
| Von Mises | 2.16 | 64.7 |
| Ins. Drucker-Prager | 2.06 | 57.3 |
| Griffith | 1.75 | 33.9 |
| Modified Griffith | 1.70 | 29.6 |
| Stassi D'Alia | 1.67 | 27.7 |
| Mohr-Coulomb | 1.65 | 25.9 |
| Murrel | 1.62 | 23.4 |
| Mogi-Coulomb | 1.59 | 21.6 |
| Modified Wiebols-Cook | 1.59 | 21.3 |
| Modified Lade | 1.56 | 19.4 |
| Cir. Drucker-Prager | 1.54 | 17.8 |
| Hoek-Brown | 1.49 | 13.6 |

[1] Meshed column indicates negative value.

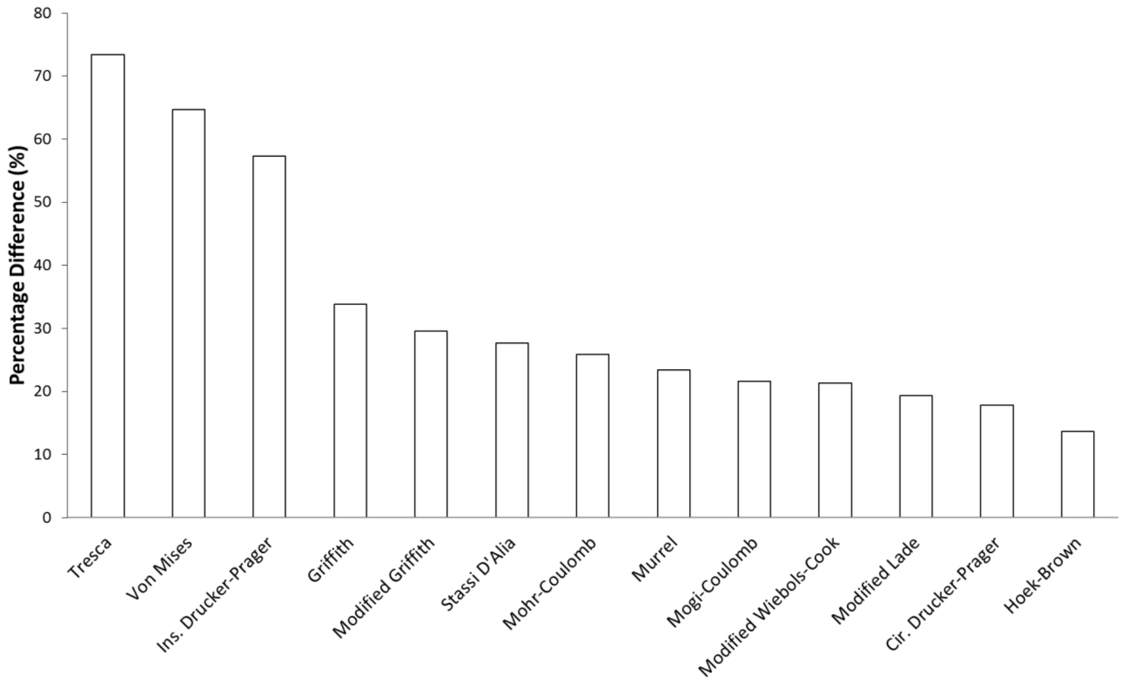


Figure 5.3. Comparison between results of predicted minimum required mud weight by different criteria and field mud weight for Etive formation (70° Inclination)

Table 5.4. Results of predicted minimum required mud weight by different criteria and difference with field mud weight for Rannoch formation (90° Inclination)

| Failure Criteria | Predicted Required Minimum Mud Weight (g/cc) | Percentage Difference with Field Mud Weight (%) |
|---|---|---|
| Ins. Drucker-Prager | 1.59 | 21.1 |
| Hoek-Brown | 1.13 | -13.5 |
| Mohr-Coulomb | 1.08 | -17.9 |
| Griffith | 1.05 | -19.6 |
| Modified Griffith | 1.04 | -20.7 |
| Mogi-Coulomb | 0.88 | -33.1 |
| Tresca | 0.84 | -35.6 |
| Modified Lade | 0.82 | -37.1 |
| Murrel | 0.62 | -52.8 |
| Cir. Drucker-Prager | 0.62 | -52.9 |
| Stassi D'Alia | 0.62 | -53.0 |
| Von Mises | 0.60 | -54.0 |
| Modified Wiebols-Cook | 0.60 | -54.2 |

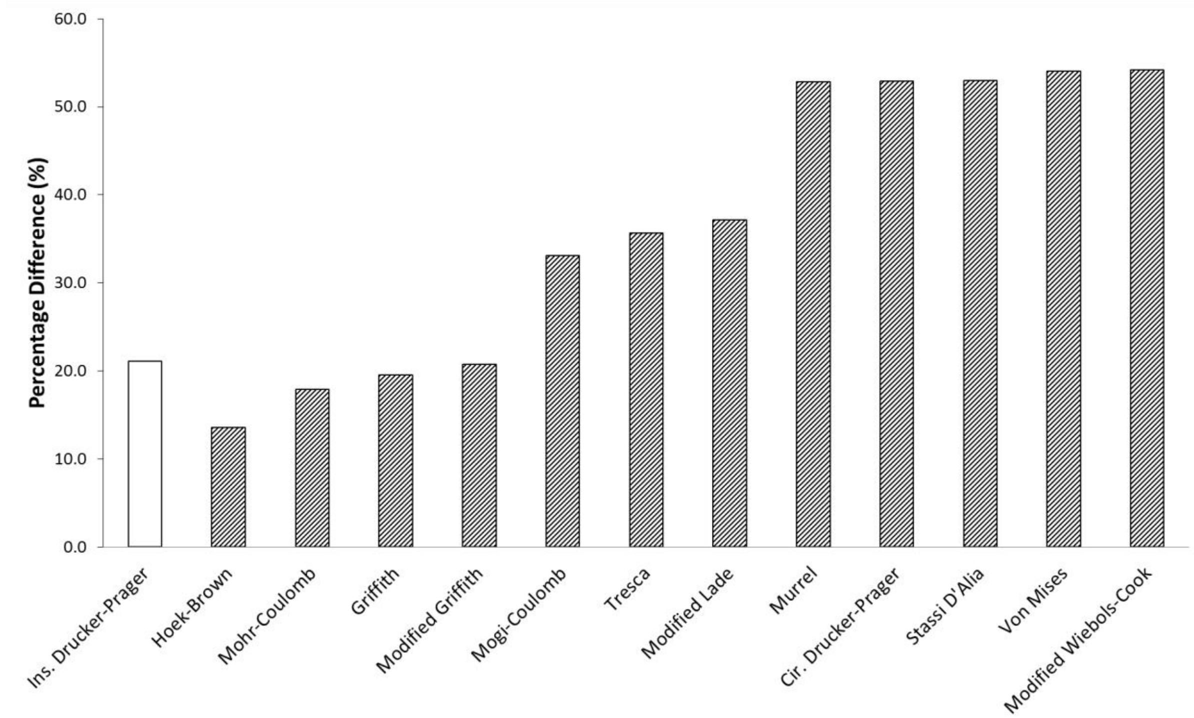


Figure 5.4. Comparison between results of predicted minimum required mud weight by different criteria and field mud weight for Rannoch formation (90° Inclination)

## 5.2. CYRUS FIELD, UK CONTINENTAL SHELF, NORTH SEA

Stress data and laboratory measured rock mechanical properties (Table 5.5) from sandstone formation in Cyrus field (UK Continental Shelf, North Sea) used as second field case study for the evaluation of failure criteria. The horizontal well was drilled in direction of maximum horizontal stress. Both horizontal and vertical well drilled successfully through sandstone formation with the mud weight of 1.17 g/cc.

Table 5.5. Field Data, Cyrus Field (Mclean & Addis., 1990)

| Formation | Rock Properties | | | Stress Data | | | | Field MW (g/cc) | Depth (m) |
|---|---|---|---|---|---|---|---|---|---|
| | $\phi$ (°) | $c$ (MPa) | ν | $\sigma_h$ (g/cc) | $\sigma_H$ (g/cc) | $\sigma_V$ (g/cc) | $p_0$ (g/cc) | | |
| Sanstone | 43.8 | 6 | 0.2 | 1.73 | 1.73 | 2.3 | 1.04 | 1.17 | 2600 |

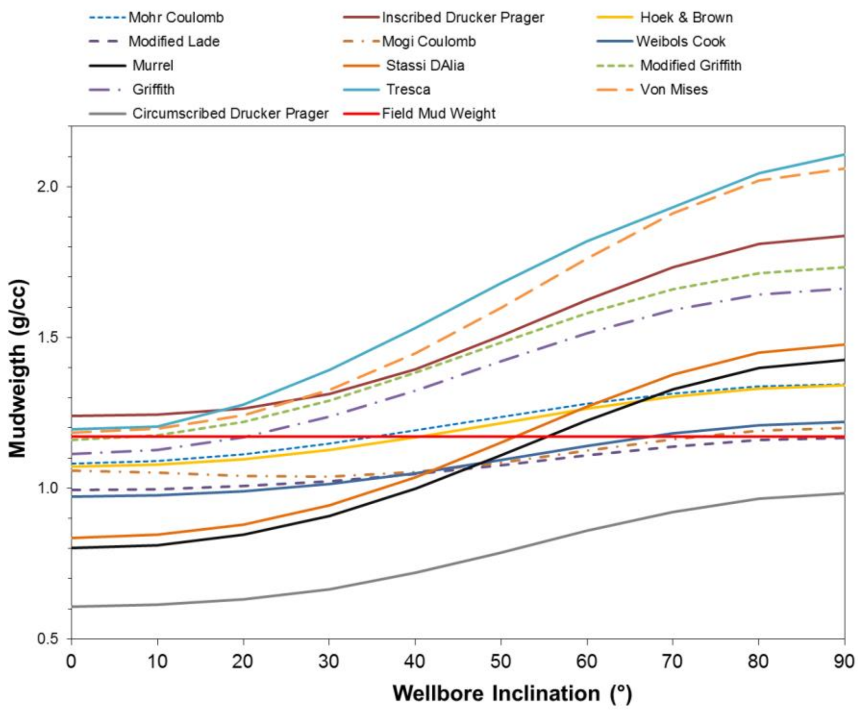


Figure 5.5. Minimum mud weight vs wellbore inclination by different failure criteria at 2600m depth and azimuth of 0° for sandstone formation, Field MW is shown with red

Table 5.4. Results of predicted minimum required mud weight by different criteria and difference with field mud weight for vertical well

| Failure Criteria | Predicted Required Minimum Mud Weight (g/cc) | Percentage Difference with Field Mud Weight (%) |
|---|---|---|
| Ins. Drucker-Prager | 1.24 | 5.9 |
| Tresca | 1.20 | 2.2 |
| Von Mises | 1.18 | 1.2 |
| Modified Griffith | 1.16 | -0.9 |
| Griffith | 1.11 | -4.9 |
| Mohr-Coulomb | 1.08 | -7.5 |
| Hoek-Brown | 1.07 | -8.4 |
| Mogi-Coulomb | 1.06 | -9.5 |
| Modified Lade | 0.99 | -15.0 |
| Modified Wiebols-Cook | 0.97 | -17.0 |
| Stassi D'Alia | 0.83 | -28.7 |
| Murrel | 0.80 | -31.6 |
| Cir. Drucker-Prager | 0.61 | -48.1 |

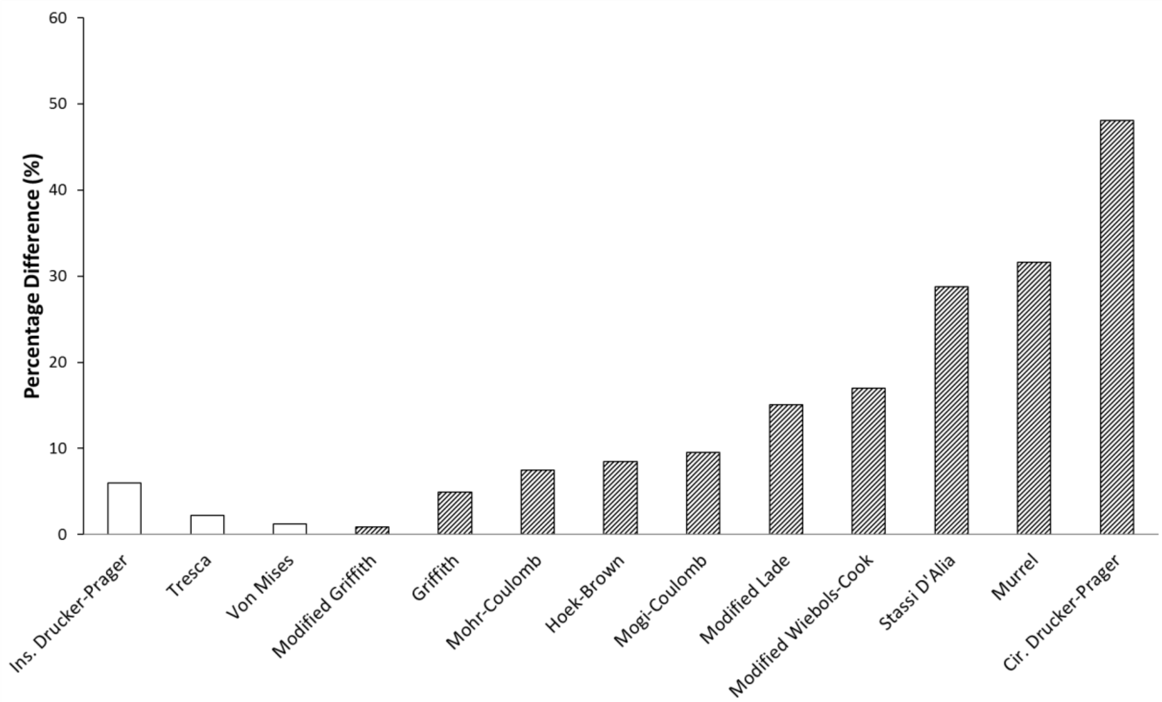


Figure 5.6. Comparison between results of predicted minimum required mud weight by different criteria and field mud weight for Vertical Well

Table 5.5. Results of predicted minimum required mud weight by different criteria and difference with field mud weight for horizontal well

| Failure Criteria | Predicted Required Minimum Mud Weight (g/cc) | Percentage Difference with Field Mud Weight (%) |
|---|---|---|
| Tresca | 2.11 | 80.1 |
| Von Mises | 2.06 | 76.1 |
| Ins. Drucker-Prager | 1.84 | 57.1 |
| Modified Griffith | 1.73 | 48.0 |
| Griffith | 1.66 | 42.0 |
| Stassi D'Alia | 1.48 | 26.1 |
| Murrel | 1.43 | 21.8 |
| Mohr-Coulomb | 1.35 | 15.0 |
| Hoek-Brown | 1.34 | 14.6 |
| Modified Wiebols-Cook | 1.22 | 4.2 |
| Mogi-Coulomb | 1.20 | 2.6 |
| Modified Lade | 1.17 | -0.2 |
| Cir. Drucker-Prager | 0.98 | -16.1 |

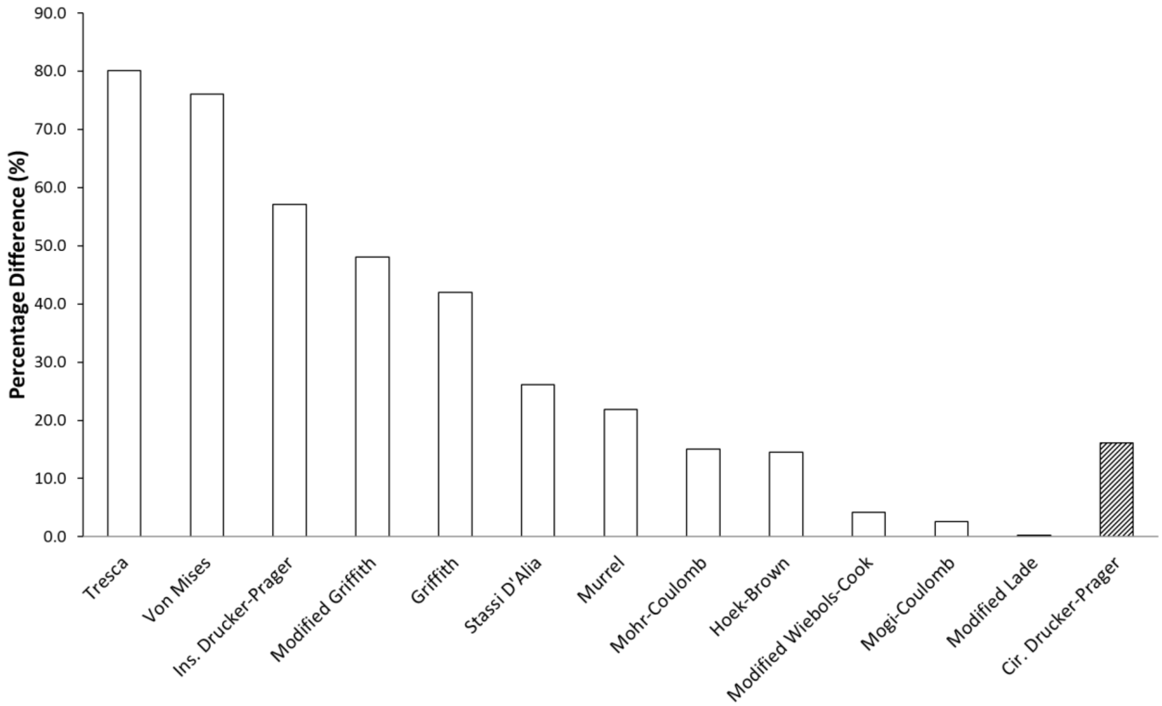


Figure 5.7. Comparison between results of predicted minimum required mud weight by different criteria and field mud weight for horizontal well

## 5.3. PAGERUNGAN GAS FIELD, NORTH OF BALI ISLAND, INDONESIA

Wellbore instabilities associated with drilling of hard and brittle Ngimbang Shale was the major challenge of developing the Pagerungan gas field in Bali Island. By determining the accurate stress field (Strike Slip faulting regime), optimization of mud weight was done which resulted in improving of drilling efficiency. Laboratory measured rock mechanical properties and stress data for Well PGA-2 are presented in table 5.6. The development PGA-2 well was drilled successfully through Ngimbang Shale with 25 degree inclination, azimuth of N47E, and mud weight of 10.6 ppg.

Table 5.6. Field Data, Well PGA-2, North Sea Oil Field (Ramos et al., 1998)

| Formation | Rock Properties | | | Stress Data | | | | Field MW (g/cc) | Depth (m) |
|---|---|---|---|---|---|---|---|---|---|
| | $\phi$ (°) | $c$ (MPa) | ν | $\sigma_h$ (g/cc) | $\sigma_H$ (g/cc) | $\sigma_V$ (g/cc) | $p_0$ (g/cc) | | |
| Sanstone | 35 | 12 | 0.3 | 2 | 2.81 | 2.3 | 1.04 | 1.27 | 1830 |

Table 5.7. Results of predicted minimum required mud weight by different criteria and difference with field mud weight for Well PGA-2

| Failure Criteria | Predicted Required Minimum Mud Weight (g/cc) | Percentage Difference with Field Mud Weight (%) |
|---|---|---|
| Ins. Drucker-Prager | 2.10 | 65.3 |
| Tresca | 2.00 | 57.7 |
| Von Mises | 1.87 | 47.5 |
| Griffith | 1.57 | 23.8 |
| Modified Griffith | 1.54 | 21.0 |
| Mohr-Coulomb | 1.44 | 13.0 |
| Hoek-Brown | 1.35 | 6.0 |
| Mogi-Coulomb | 1.19 | -6.2 |
| Modified Wiebols-Cook | 1.14 | -10.6 |
| Modified Lade | 1.13 | -11.2 |
| Stassi D'Alia | 1.12 | -12.1 |
| Murrel | 1.03 | -19.2 |
| Cir. Drucker-Prager | 0.87 | -31.7 |

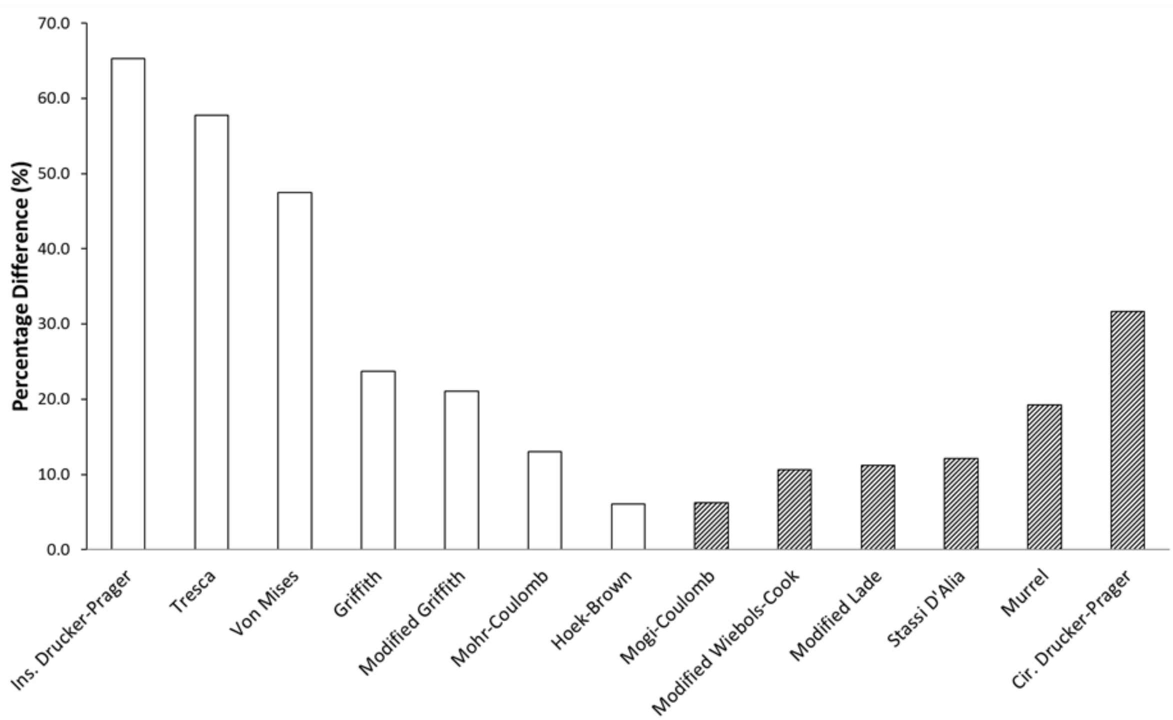


Figure 5.8. Comparison between results of predicted minimum required mud weight by different criteria and field mud weight for Well PGA-2

# 6. DISCUSSION

According to the results of the statistical analysis, rock mechanical properties as a main category of variables can significantly change the distribution of results for the minimum required mud weight by different rock failure criteria. Major difference in the frictional angle and UCS may cause different rock failure criteria to be in the higher and lower boundaries of the results. The Mohr-Coulomb criterion always estimates the higher minimum required mud weight than actual polyaxial failure criteria because of neglecting the effect of intermediate principal stress. For the high strength formation, the results of Mohr-Coulomb criterion approach upper bounds of results which show it might not be an accurate criterion for failure analysis of hard lithology. The Mohr-Coulomb criterion is mainly represents the middle range of results. The results of the polyaxial Mogi-Coulomb criterion do not show a major difference when varying rock mechanical properties and are always between middle range to lower range of the results.

The Tresca criterion is mainly representing the highest boundary of the results. Since linear the Tresca criterion is only dependent on a single variable (UCS) and also does not include the effect of intermediate principal stress, it cannot give an accurate description of stress condition at breakout failure around a wellbore. The results of the Inscribed Drucker-Prager replaced the Tresca as the highest boundary for high strength formations. The Von Mises criterion predicts high values for the minimum required mud weight since this criterion does not change results with changing differential stress around the borehole due to its symmetrical properties of failure. Although intermediate principal stress does effect rock strength, it is not significant as the other principal stresses. Dependency on a single variable is other shortcoming of this criterion for accurate description of stress condition at failure and estimation of the minimum required mud weight. The results of Von Mises criterion approached the middle range of results for the high strength formations.

The Inscribed Drucker-Prager criterion estimates the high range of minimum required mud weight compared to other rock failure criteria because of underestimation of rock strength. The results of the Inscribed Drucker-Prager are higher than the Tresca and the Von

Mises for the high strength formation. The Circumscribed Drucker-Prager is mainly estimating the lower minimum required mud weight than other rock failure criteria because of overestimation of rock strength. The results of the Circumscribed Drucker-Prager are significantly lower than other failure criteria for the weaker formation with high internal angle of friction. For the hard formation, the results of the Circumscribed Drucker-Prager show close similarity to the Midfield Lade, the Mogi-Coulomb, and the Modified Wiebols-Cook results especially for the horizontal borehole. In this situation, the results of the Murrel criterion replaced the Circumscribed Drucker-Prager as the lowest boundary of results. The results of the Hoek-Brown criterion mainly have minimum difference between the vertical and horizontal borehole. Range of results for the Hoek-Brown criterion is remarkably depends on the magnitude of material constant, $m$ for different lithology.

The results of the Modified Lade and the Modified Wiebols-Cook show very close similarity to the results of the Mogi-Coulomb criterion for all cases of statistical analysis. These three polyaxial failure criteria typically have small difference between results for vertical and horizontal borehole. Also, variation of rock mechanical properties does not change the range of results by these three criteria. These three criteria potentially can give more accurate description of rock at failure because of considering the effect of intermediate principal stress on rock strength and good fitting of polyaxial test data for different types of rock.

Range of results for the Griffith criterion is changing by variation in compressive strength of rock (UCS) since the tensile component in this criterion determines from a constant ratio with UCS. For the Modified Griffith criterion, variation of rock mechanical properties has more influence on the range of results for minimum required mud weight because the tensile component in the Modified Griffith is a function of both UCS and internal angle of friction. For the hard formations with low internal angle of friction, the Modified Griffith criterion represents the lower range of the results. For weak formations with high internal angle of friction, the Modified Griffith criterion predicts the higher range of results for the minimum required mud weight.

Variation of rock mechanical properties affects the results of the Murrel and the Stassi d'Alia more than other criteria. The Murrel criterion estimates the lowest boundary of results for hard formations with low internal angle of friction since the tensile component in this criterion only depends on uniaxial compressive strength (UCS). In general, increasing UCS caused the results of both the Murrel and the Stassi d'Alia approach the lower range of results for the minimum required mud weight. For medium strength and weak formations, the results of the Murrel and the Stassi d'Alia criteria represent the middle range of results.

In general, distribution of the result in normally pressurized zone (lower depth) is higher than over pressurized zone (deeper depth). Furthermore, differences between the results for the vertical and horizontal boreholes by different failure criteria are typically higher in normally pressurized zone rather than over pressurized zone of deeper depth. Based on the results of all cases, increasing azimuth usually caused that distribution of results to be lower for the horizontal borehole.

According to the results of field cases, the Mogi-Coulomb, the Modified Lade, and the Modified Wiebols-Cook predict minimum required mud weight which has close similarity to the field mud weight used to successfully drill the borehole. For some cases, the lower mud weight can be potentially used because the lower mud weight estimated by those polyaxial failure criteria which give better description of stress condition at failure. Other than those three criteria, the Hoek-Brown criterion mainly estimates close results to the field mud weight. Results of field cases verified the results of statistical analysis about the different distribution and boundaries of results due to variation of rock mechanical properties. There was a different distribution of results for the Rannoch formation of the North Sea Oil Field. Since the Rannoch formation has the highest UCS between all cases in statistical analysis and field cases, different failure criteria represent the lower boundaries of results. In this situation, Modified Wiebols-Cook and Von Mises criteria predict the lower bound of results.

# 7. CONCLUSIONS

Based on the results of statistical analysis, rock mechanical properties should be considered as key factor in the process of selecting appropriate rock failure criterion for wellbore stability analysis. A failure criterion might give comparable results with other criteria in one condition but estimates very high or very low results for the minimum required mud weight in other situation due to variation of rock mechanical properties. The results of the Murrel, the Stassi d'Alia, the Modified Griffith, and the Circumscribed Drucker-Prager criteria show high sensitivity to variation of rock mechanical properties.

The Tresca, the Von Mises, and the Inscribed Drucker-Prager estimate the higher bounds for the minimum required mud weight for all cases. Although the Circumscribed Drucker-Prager usually predicts the lower bound for the minimum required mud weight, its results are in the middle range for the hard formations with low internal angle of friction. The Modified Lade, the Modified Wiebols-Cook and the Mogi-Coulomb give similar results for three lithology studied so these failure criteria may be used interchangeably without altering the results.

The results of field case studies show that the Mogi-Coulomb, the Modified Lade, and the Modified Wiebols-Cook criteria predict close results to the field mud weight used to successfully drill the borehole. For some field cases, lower mud weight can be potentially used because the lower results estimated by those three polyaxial failure criteria which give more accurate description of stress condition at failure. The Hoek-Brown criterion mainly estimates close but higher results to the field mud weight for the studied cases. Since the results of this criterion are mainly dependent to material parameter, $m$ it might give unrealistic results in other situation. The results of field case study are comparable with the results of statistical analysis about the different distribution and boundaries of results for the minimum required mud weight due to variation of rock mechanical properties.

# APPENDIX

## DERIVATION OF ROCK FAILURE CRITERIA

This appendix explains and derives the failure criteria investigated in chapter 2.1:

### 1. MOHR-COULOMB

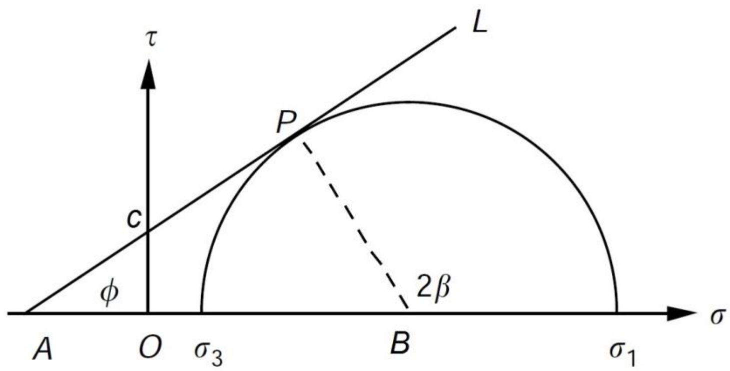


Figure A-1. Mohr-Coulomb failure envelope in $\tau - \sigma$

Governing equation for Mohr-Coulomb failure criterion is function of normal stress components and rock mechanical properties (Eq A.1). Based on the Mohr-Coulomb theory, failure in compression will happen at point P where shear stress on specific planes reaches value that is sufficient to overcome both the natural cohesion of rock and frictional force that oppose along the failure plane:

$$\tau = c + \mu\sigma \tag{A.1}$$

The parameter ***c*** is known as cohesion and parameter ***μ*** is the coefficient of internal angle of friction. The point P which is tangential point of Mohr circle and failure envelope represents state of stress on the plane of failure. By using basic trigonometric, the relation between angle of internal friction and angle β which is orientation of failure plane in to direction of $\sigma_1$ could be derived as shown here:

$$2\beta = 180^\circ - \angle ABP \tag{A.2}$$

$$\angle ABP = 180^\circ - \angle BPA - \angle PAB \tag{A.3}$$

As P is tangential point of the circle and the failure line, line BP is perpendicular to failure line (AL):

$$\angle CBA = 90^\circ \tag{A.4}$$

Finally, following relation derived based on (A.2), (A.3), and (A.4):

$$2\beta = 90^\circ + \phi \quad or \quad \beta = 45^\circ + \frac{1}{2}\phi \tag{A.5}$$

The Mohr-Coulomb can also be expressed in term of principal stresses which is presented here:

$$|BP| = (|AO| + |OB|)\sin\phi \tag{A.6}$$

This above relation can be written in following form:

$$\frac{1}{2}(\sigma_1 - \sigma_3) = c\cos\phi + \frac{1}{2}(\sigma_1 + \sigma_3)\sin\phi \tag{A.7}$$

The mean normal stress ($\sigma_m$) and maximum shear stress ($\tau_m$) on the figure is:

$$\sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3), \qquad \tau_m = \frac{1}{2}(\sigma_1 - \sigma_3) \tag{A.8}$$

Then Mohr-Coulomb failure criterion can be presented in following way:

$$\tau_m = c\cos\phi + \sigma_m \sin\phi \tag{A.9}$$

By changing order of (A.7), we will have Mohr-Coulomb failure criteria in form of principal stresses:

$$\sigma_1 = 2c\frac{\cos\phi}{1-\sin\phi} + \sigma_3\frac{1+\sin\phi}{1-\sin\phi} \tag{A.10}$$

This can be simplified to following form:

$$\sigma_1 = \mathrm{C}_0 + \sigma_3 q \tag{A.11}$$

The $\mathrm{C}_0$ is uniaxial compressive strength (UCS) and $q$ is the flow factor which is function of internal angle of friction. The $\mathrm{C}_0$ and $q$ parameters can be found from the results of triaxial tests.

$$\mathrm{C}_0 = 2c\frac{\cos\phi}{1-\sin\phi}, \tag{A.12}$$

$$q = \frac{1+\sin\phi}{1-\sin\phi} \tag{A.13}$$

## 2. MOGI-COULOMB

Mogi (1971) verified the effects of the intermediate principal stress on the strength of couple of different rock types based on the results of the true triaxial test. Mogi found out that mean normal stress that opposes creation of fracture is $\sigma_{m,2}$, rather than octahedral normal stress, $\sigma_{oct}$ .

$$\tau_{oct} = f(\sigma_{m,2}) \tag{A.14}$$

Based on Mogi's assumption and the polyaxial test data provided by Takahashi and Koide (1989), Al-Ajmi and Zimmerman (2005) found linear relation that could give a good fit with test results in $\tau_{oct} - \sigma_{m,2}$ space:

$$\tau_{oct} = a + b\sigma_{m,2} \tag{A.15}$$

The parameter $a$ is intersection of line on the $\tau_{oct}$ axis and $b$ is the inclination of the envelope in $\tau_{oct} - \sigma_{m,2}$ space. Al-Ajmi and Zimmerman tested new developed linear criterion for polyaxial test data from eight different rock types and the results verified the Mogi failure based on triaxial test data correlates well with the polyaxial test data. Therefore the Mogi Coulomb failure parameters $a$ and $b$ can be evaluated based on the triaxial test result.

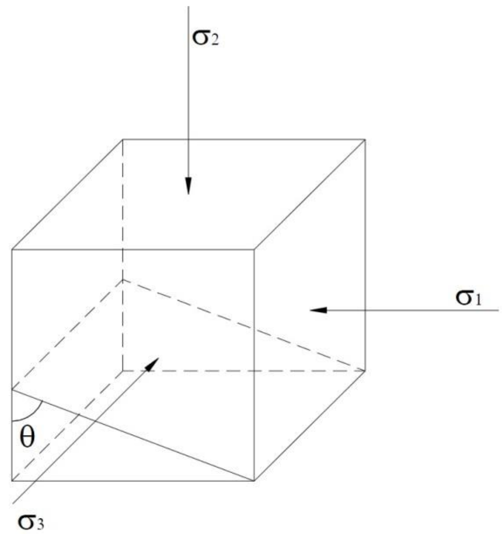


Figure A-2. Principal Stresses

The direction which plane will be equally inclined to principal axes is:

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{\sqrt{3}} \tag{A.16}$$

The plane is called octahedral plane because it is parallel to face of an octahedron with vertices on the principal axes. The normal and shear stress acting on this plane called the octahedral shear stress ($\tau_{oct}$) and octahedral normal stress ($\sigma_{oct}$). The stresses σ and τ in a general direction $\lambda_1$, $\lambda_2$, and $\lambda_3$ will be determined by following relation (Fjaer et al., 2008):

$$\lambda_1{}^2\sigma_1 + \lambda_2{}^2\sigma_2 + \lambda_3{}^2\sigma_3 = \sigma \tag{A.17}$$

$$\lambda_1{}^2\sigma_1{}^2 + \lambda_2{}^2\sigma_2{}^2 + \lambda_3{}^2\sigma_3{}^2 = \sigma^2 + \tau^2 \tag{A.18}$$

By substituting (A.16) into (A.17) and (A.18), octahedral normal and shear stress will be:

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{A.19}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2} \qquad (A.20)$$

Considering conventional triaxial test ($\sigma_2$=$\sigma_3$), octahedral shear stress will reduce to:

$$\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2} = \frac{\sqrt{2}}{3}(\sigma_1-\sigma_3) \qquad (A.21)$$

The substituting of (A.21) into (A.15) will result in:

$$\frac{\sqrt{2}}{3}(\sigma_1-\sigma_3) = a + b\sigma_{m,2} \qquad (A.22)$$

Rearranging (A.22) will change the linear Mogi criterion to a new form:

$$\frac{1}{2}(\sigma_1-\sigma_3) = (a\frac{3}{2\sqrt{2}}) + (b\frac{3}{2\sqrt{2}})\sigma_{m,2} \qquad (A.23)$$

Considering maximum shear stress definition in A.8, the relation A.23 can be written in following form:

$$\tau_{max} = (a\frac{3}{2\sqrt{2}}) + (b\frac{3}{2\sqrt{2}})\sigma_{m,2} \qquad (A.24)$$

If we compare the Mohr-Coulomb failure criterion given in A.9 with A.24, the linear Mogi-Coulomb criterion parameters $a$ and $b$ can be determined by following relations:

$$a = \frac{2\sqrt{2}}{3}\, c\, \cos\phi \qquad (A.25)$$

$$b = \frac{2\sqrt{2}}{3}\sin\phi \qquad (A.26)$$

The parameter $b$ represents the internal friction, while the parameter $a$ is related to the both cohesion and internal friction of rock. Using relation A.12 and A.13, the linear Mogi-Coulomb failure criterion parameters $a$ and $b$ can be identified based on the Mohr-Coulomb criteria parameters (q, $C_0$):

$$a = \frac{2\sqrt{2}}{3}\frac{C_0}{q+1} \tag{A.27}$$

$$b = \frac{2\sqrt{2}}{3}\frac{q-1}{q+1} \tag{A.28}$$

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# VITA

Reza Rahimi was born on September 21, 1986. Reza entered Petroleum University of Technology in Abadan, Iran on 2005 and completed his Bachelor's degree in 2009. He has done two summer internship with Iranian Offshore Oil Company (IOOC) and National Iranian Oil Company (NIOC) on 2007 and 2008. After finishing his Bachelor, he was employed by PETROPRAS Oil and Gas Institute (POGI) as Research Petroleum Engineer. Then he joint MAPSA ENG Co. as Drilling Engineer for the exploratory wells drilled in the Golshan and Ferdows offshore gas fields in Persian Gulf. He was involved in well planning and drilling program. Later on, he was hired by PPL Ltd as Field Drilling Engineer on Jack up Rig for the wells drilled in the phase 12 of South Pars gas field in Persian Gulf. He was in charge of supervision on daily drilling activities, providing daily drilling report, and management of field operation. In August 2012, he entered the Graduate School of Missouri University of Science and Technology as a Master's student in Petroleum Engineering program. While working on his Master, Reza served as a Teaching Assistant for the Drilling & Well Design class. Reza completed his MS in Petroleum Engineering in May, 2014.