On finding the expression for the elasticity tensor linking tension and torsion.

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The research is devoted to determination of the elastic moduli of curved rods.

The main idea is to have the angular momentum and the linear momentum received according to the rods' theory in line with ones received from the modeling in 3d.

$$\rho_0(u\cdot\Theta_1+\Theta_2\cdot\psi)=\int\rho a\times u_{(3)}dxdy \qquad \rho_0(u+\Theta_1\cdot\psi)=\int\rho u_{(3)}dxdy$$

I General relations of the rods theory.

$$\rho_0 u(\underline{e},\underline{k})=\frac{1}{2}\underline{e}\cdot\underline{\underline{A}}\cdot\underline{e}+\underline{e}\cdot\underline{\underline{B}}\cdot\underline{k}+\frac{1}{2}\underline{k}\cdot\underline{\underline{C}}\cdot\underline{k}$$

$$\frac{\partial\rho_0 u(\underline{e},\underline{k})}{\partial\underline{e}}=\underline{F} \qquad \frac{\partial\rho_0 u(\underline{e},\underline{k})}{\partial\underline{k}}=\underline{M} \qquad \underline{e}=u'+t\times\psi \qquad \underline{k}=\psi'$$

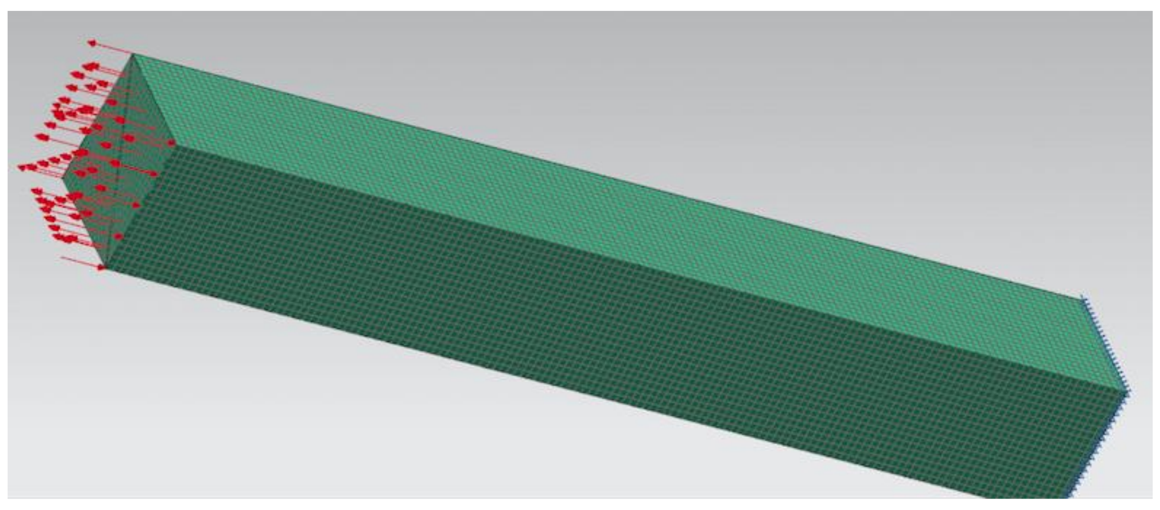
There are several elasticity tensors describing deformation of thin rods, which are responsible for:

the rod tension and laminated shift: $\mathbf{A}=A_1\mathbf{d}_1\mathbf{d}_1+A_2\mathbf{d}_2\mathbf{d}_2+A_3\mathbf{d}_3\mathbf{d}_3$;

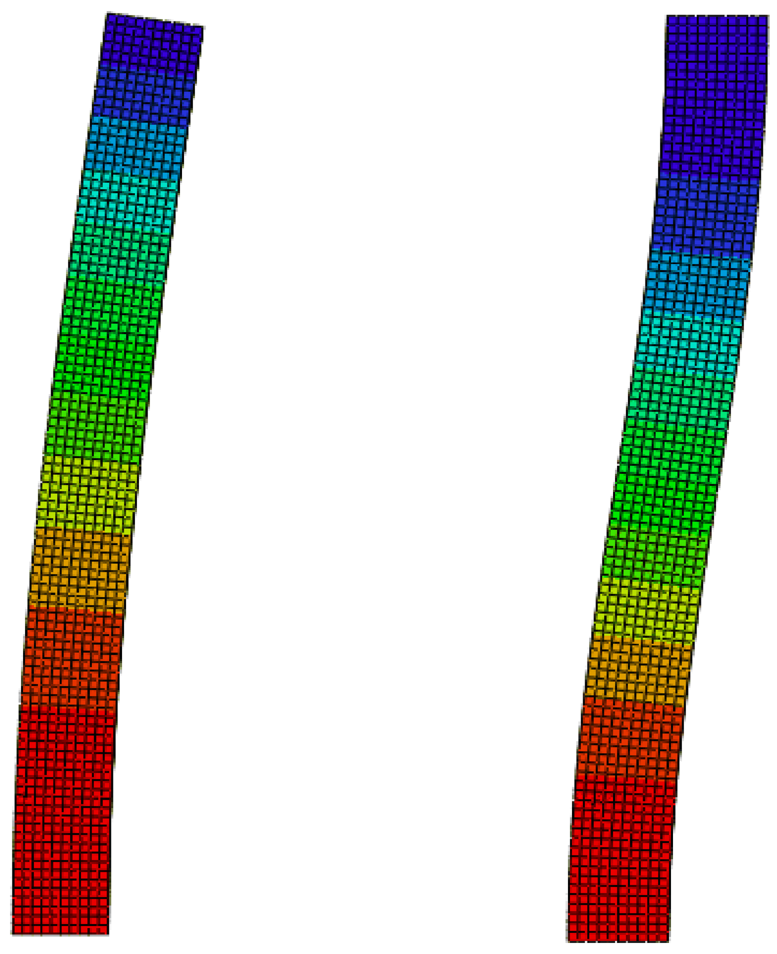
the rod bending and torsion: $\mathbf{C}=C_1\mathbf{d}_1\mathbf{d}_1+C_2\mathbf{d}_2\mathbf{d}_2+C_3\mathbf{d}_3\mathbf{d}_3$.

II Verification for the rod with well-known elastic moduli.

In this stage it is necessary to check the results by calculation the measurement error. It depends on the amount of the finite elements in the mesh. The measurement error = 0.03, so the results derived from the modeling are considered to be accurate enough.



Straight rod in 3d with a momentum load applied



1) Free edge 2) Hinge

Rods with different boundary conditions and the same distributed force applied .

III The expression for the shear component of tensor and corresponding shear coefficients

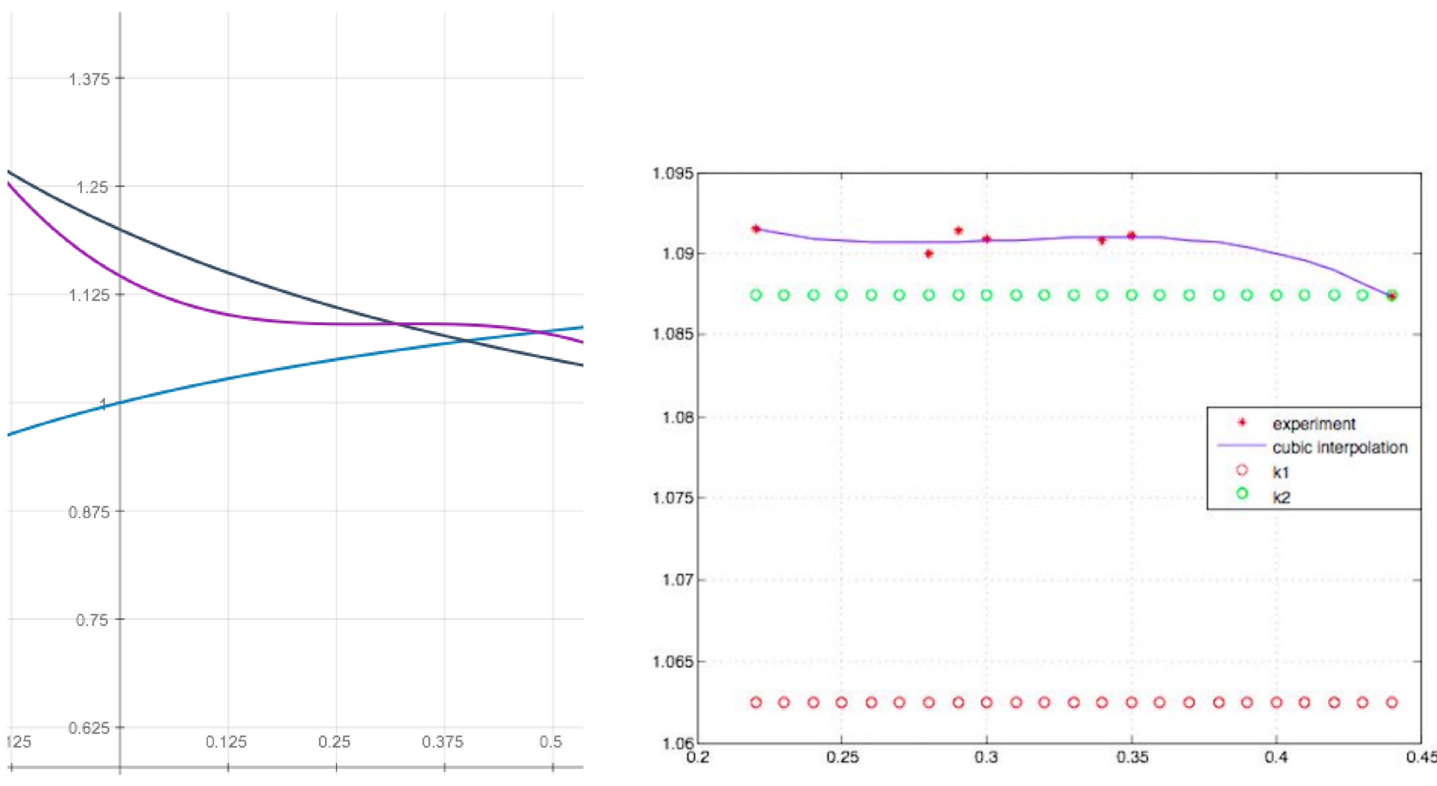
The components of this tensor may be derived from the experiments with straight rods.

$$A_x=\frac{Fbl}{2u_2(3-b)-(3-2b)u_1} \qquad k_x=\frac{A_x}{ES}$$

Here b is the non-dimensional length coefficient, F-the force applied, l-length of the rod, u_1 and u_2 are the displacements for the 1st and the 2d boundary conditions respectively.

$k_x(\nu)=-1.98\nu^3+1.83\nu^2-0.55\nu+1.15$

k_1, k_2– shear coefficients, obtained by Y.M. Gavrilov. (E.I.Grigolyuk, I.T. Selezov, p.51)



The plots of shear coefficients versus poisson coefficients

IV Results.

The methodology of determination the elastic moduli in the linear rods theory was tested, the measurement error was derived, and the dependence of shear coefficient on poisson coefficient was obtained.

This work is the basement for the future investigation on this subject.

In perspective, the research will be devoted to the problem of finding the expression for the third elasticity tensor, which components represent the modules of elasticity, evaluating the connection between tension and torsion.