Mathematical methods for modelling of environmental processes

To predict environmental processes like the spreading of pollutants often the well known advection diffusion equation is employed. A solution can be found by solving this equation numerically. Alternatively one can derive a stochastic differential equation for the irregular behavior of one small particle of the pollutant considered. By simulating the behavior of many different particles the spreading of the pollutant can be modeled. An advantage of this Lagrangian approach is that it can deal easily with steep concentration gradients. Furthermore, particle models are inherently suited for parallel processing.

The lectures will discuss a number of mathematical aspects of this approach. The various concepts will be illustrated with test cases and applications to various environmental problems.

Lecture 1 (3 hours): Introduction to stochastic processes and stochastic differential equations

Some basic results from probability theory. Wiener processes and White noise, Definition of a stochastic differential equation, discussion of the different stochastic calculus that can be used (Ito, Stratonovitz, Backward-Ito)

Lecture 2 (3 hours): Modelling environmental transport processes using stochastic differential equations.

The advection diffusion equation for describing transport processes: Derivation and analysis. The Fokker-Planck equation to describe the evolution of a probability function of a process generated by a stochastic differential equation and the connection to the advection diffusion equation for pollution transport. Stochastic particle models for describing the dynamic behavior of individual particles of the pollutant.

Lecture 3 (3 hours): Particle models for pollution transport and reverse time diffusion.

Various particle models for pollution transport: random walk models, models with decay, models with non-Fickian diffusion, random flight models. The concept of reverse time diffusion. Applications to coastal sea and ocean transport problems.

Lecture 4 (3 hours): Numerical treatment of stochastic differential equations

Numerical schemes for stochastic differential equations. Strong en weak order of convergence of numerical schemes and adaptive time stepping methods. Applications to coastal sea and ocean transport problems.