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INTRODUCING A 2D EXPERIMENTAL SET-UP FOR MODELING HYDRAULIC FRACTURING IN SYNTHETIC ROCK-LIKE MATERIALS

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ABSTRACT

Crack initiation and propagation is a three-dimensional process. Most of the analytical solutions (such as PKN and KGD models) and numerical models treat crack propagation as a two-dimensional (2D) process. Yet, there is no experimental study, which can provide a one to one comparison in 2D to validate these kinds of models. The 2D experimental set-up equipped with a high-speed camera provides continuous video record and measurement of fracture path. A Speckle Pattern is applied in order to accurately measure surface deformation with Digital Image Correlation (DIC). A transparent material is used in order to have a direct viewing of fracture growth. The results provide information about the breakdown pressure, fracture growth direction, width and fracture speed.

KEYWORDS

Rock-Like Materials, Hydraulic Fracturing, Experimental set-up, Crack initiation, transparent material

INTRODUCTION

Hydraulic fracturing has been used in different applications since 1950.There are still many open questions and uncertainties related to hydraulic fracturing. Observing the fracture geometry in field treatments is almost impossible, except in special tests with extensive seismic monitoring (Abe *et al.*, 1983; Vinegar *et al.*, 1992), even in those cases it is believed (de Pater *et al.*, 1994) the data interpretation needs to be more developed. To better understand the behavior of rock during a hydraulic fracturing treatment, numerous studies have been undertaken and several physical models developed to investigate rock behavior during the injection. Laboratory tests should, therefore, serve as benchmarks for numerical simulations.

In this study, an innovative two-dimensional set-up is introduced for conducting hydraulic fracturing on low-strength rock-like materials.

1. BACKGROUND

There are three different approaches for simulating crack propagation. (1) Analytical, (2) Numerical and (3) physical models are introduced and related literature are summarized.

1.1. Analytical Models

Two types of analytical approaches are commonly used in 2D fracture propagation simulation: one is presented by Perkins and Kern (PK) (Perkins and Kern, 1961) and modified by Nordgren (1972) known as PKN (Perkins–Kern–Nordgren) and the other one is given by Geertsma and de Klerk based on earlier works by Khristianovic and Zheltov (1955) and Geertsma and De Klerk (1969) known as GDK (Geertsma–de Klerk– Khristianovic). In PKN, the cross-section of the fracture in the vertical plane, perpendicular to the long axis of the fracture, maintains an elliptical configuration and plane strain condition exists in the vertical plane. On the other hand, the GDK model approach presumes an approximately elliptical configuration in the horizontal plane and a rectangular shape in the vertical plane and plane strain conditions in the horizontal plane. A schematic illustration of the two models is given in Figure 1. Further common assumptions of the two models are the constant fracture height independent of the fracture length and the zero net pressure at the fracture tip. It is generally accepted that the PKN-model is most appropriate when length to height ratios are much larger than one, while the KGD-model is typically used for small length to height ratios (less than one) (Fjar *et al.*, 2008). This implies that the PKN-model is more used in conventional HF modelling where the fracture is long compared to the fracture height. On the other hand, for open-hole stress tests where the fracture is normally short compared to the height, the KGD-model should give a better approximation.

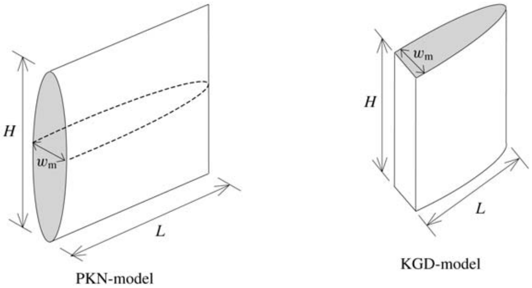

Figure 1. Illustration of fracture shapes for the PKN and KGD models (Fjar et al., 2008)

According to the PKN solution (Perkins and Kern, 1961), the net pressure, P_{net} (Pa), can be approximated using the following expression (Itasca, 2013).

$$P_{net} = \left[\frac{16\mu q E^3}{\left(1-\nu^2\right)^3 \pi H^4}\right]^{\frac{1}{4}} \tag{1}$$

The net pressure means to subtract the in-situ stresses from the injection fluid pressure. In any vertical elliptical cross-section perpendicular to the direction of propagation, the fluid pressure, p, is constant (i.e., no vertical pressure drop).

Also, the expression for the width, w(x) (m) in the middle of the fracture is:

$$w(x) = 3\left[\frac{\mu q\left(1-\nu^2\right)\left(L-x\right)}{E} L\right]^{\frac{1}{4}} \tag{2}$$

where E (Pa) is Young's modulus, v is the Poisson's ratio, q (m^3/s) is the injection rate, μ (Pa.sec) is the viscosity of the injected fluid, and L (m) and H (m) are the length and height of the fracture, respectively, as illustrated in Figure 2.

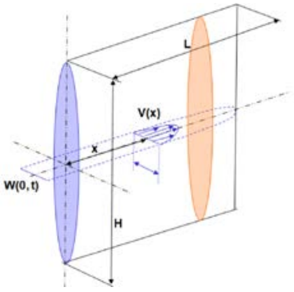

Figure 2 Geometry of the PKN model (Itasca, 2013)

Although these analytical models have been globally accepted and used in different commercial codes, it should be noted that these models have some simplification assumptions and the results might have low precision.

Thus, it is worth to introduce a physical model with almost the same geometrical assumptions. Then, the outcome of the physical model will be compared with the well-developed formulations such as net pressure or fracture width.

1.2. Numerical models

The rock masses contain several joints and natural fractures which influence the hydraulic fracture treatments. A numerical model must represent two types of mechanical behaviour in a discontinuous system such as naturally fractured reservoirs. (1) The behaviour of intact rock to represent the behaviour of solid material (matrix) that constitutes the particles or blocks in the discontinuous system. (2) The behaviour of the discontinuities to recognize the existence of contacts or interfaces between discrete bodies that comprise the rock system. (Hamidi and Mortazavi, 2014). Discrete Element Method (DEM) is the most popular method for simulating the mechanical responses of a dis-continuum medium. The numerical model which is going to be discussed in this study will focus mostly on the crack initiation and propagation in intact rock by DEM. Hamidi *et al.* (2016) wrote a comprehensive review of current advances in DEM models for simulating hydraulic fracturing. A short summary of the paper is provided in this section.

DEM is a numerical solution used to describe the mechanical behaviour of discontinuous bodies. DEM was developed (Cundall, 1971b; Cundall, 1971a) for the analysis of rock mechanic problems using deformable polygonal-shaped blocks and then applied to soils (Cundall and Strack, 1979). This led to the development of Itasca's UDEC (Universal Distinct Element Code) and 3DEC (Three-Dimensional Distinct Element Code) software. PFC is a simplified implementation of the DEM because it utilizes rigid disks (PFC2D) or spherical particles (PFC3D) to extensively simplify contact detection between elements for faster model solutions.

DEM has been used as a tool for simulating fracturing process in intact rocks since 1989 by Lorig and Cundall (1989). In UDEC Code Brostow *et al.* (1978); Finney (1979); Medvedev (1986) simulated intact material by Voronoi polygons assemblages (or tessellation). A fracture is assumed to be formed when the stress level at the interface between block exceeds a threshold value either in tension (Jkn) or in shear (Jks). Nasehi and Mortazavi (2013) employed the same method in UDEC for simulating the crack propagation both in intact rock and naturally fractured zone. The numerical approach has extended into 3D by Hamidi and Mortazavi (2014). The geometry of the 3D model was created by importing tetrahedron tessellation in Three-dimensional Distinct Element Code (3DEC). The drawback of using tetrahedron blocks is the higher possibility of interlocking. This interlocking may lead to increase the stiffness in contacts. Recently, Hamidi *et al.* (2016) suggested a 3D Voronoi tessellation to avoid this problem.

Many other studies such as Pournin and Liebling (2005) generalized the DEM to sphero-polyhedral particles (DSEM) for modelling 3D particles with complex shapes. Galindo Torres and Muñoz Castaño (2007) introduced a DEM model which represents intact rock as an assembly of Voronoi polygons jointed by beams. Alonso-Marroquín (2008) modified this method by introducing a multi-contact approach in 2D for modelling non-convex shapes. This method has been extended to 3D by Galindo Torres *et al*. (2009), which is published as an open source code known as MechSys (Galindo Torres *et al.,* 2012). Behraftar *et al.* (2017) used MechSys to simulate crack propagation in Crack Chevron Notch Brazilian Disc (CCNBD) (Ouchterlony, 1988). For simulating the fracking, a further analysis in a combination of DSEM and Lattice Boltzmann Method (LBM) has been conducted by Galindo Torres (2013) to represent more realistic coupled behaviour. Gerolymatou *et al.* (2015) have recently used the same code to investigate the effect of pre-existing discontinuity on hydraulic stimulation.

In all the DEM simulations, the micromechanical parameters (the values of Jkn and Jks) need to be evaluated based on laboratory experiments. Although there is an enormous difference in the scale of fractures in laboratory tests (crack) and in field applications (fracture), as de Pater *et al.* (1994) declared "*a numerical model should at least be capable of describing model tests with the appropriate boundary conditions*". Thus, the physical models in laboratories could validate the numerical models. The validated models can be used for simulating hydraulic fracturing on a larger scale.

1.3. Physical models

Most of these Physical models are largely conducted on cubic blocks with three-dimensional loading condition using opaque samples. In these models, induced fracture geometry is recorded in two ways. (1) Destructive: by cutting the sample after the test and (2) Non-Destructive: by using acoustic emission or seismic monitoring systems. While these models are useful, destructive sampling only shows the final result and the non-destructive options provide insufficient details. Thus, it was impossible to track the crack initiation and measure crack propagation geometry parameters directly during the injection experiments. The contribution of this study is to address this problem by providing a two-dimensional experimental set-up for investigating crack initiation and propagation in a transparent synthetic material.

Synthetic materials have been used for simulating the hydraulic fracturing since the 1950s, in particular to ensure reproducibility of results and to maintain uniformity and homogeneity (Hubbert and Willis, 1957; Haimson and Fairhurst, 1969; Clifton *et al.*, 1976; Papadopoulos *et al.*, 1983; Weijers and de Pater, 1992; Bunger *et al.*, 2005a; Jeffrey *et al.*, 2015; Xu *et al.*, 2015). Laboratory experiments on hydraulic fracturing in transparent materials have been conducted allowing direct detailed visualization of the initiation and early growth (Rummel, 1987; Takada, 1990; Bunger *et al.*, 2004; Bunger, 2005; Bunger *et al.*, 2005c; b; Wu, 2006; Wu *et al.*, 2007; Wu *et al.*, 2008; Bunger *et al.*, 2011; Frash, 2012; Bunger *et al.*, 2013; Frash, 2014; Frash *et al.*, 2014; Kovalyshen *et al.*, 2014). The visualization in real time of the developing geometry of the fracture and the direction of fracture propagation are the two main advantages of transparent materials.

The growing fracture is monitored using a video camera and backlight. The images of the growing fracture are not only useful for measuring the fracture radius, but they also enable estimation of the full-field fracture opening. The intensity of light is decreased by an order of magnitude when it passes through a fluid layer with a specific thickness (Bunger *et al.*, 2004; Bunger *et al.*, 2005a; Bunger *et al.*, 2005c; b; Bunger, 2006; Kovalyshen *et al.*, 2014). Validation of the method was presented through experiments performed wherein hydraulic fractures were driven beneath a thin plate so that the deflection of the plate, as measured by an LVDT, corresponds to the thickness of the fluid layer and could, therefore, be directly compared with photometric estimates (Bunger, 2006). Later Kovalyshen *et al.* (2014) conducted a comparison between photometric and the ultrasound method in a glass block. They found that these two methods are in a good agreement from a practical standpoint for monitoring the location of the crack front and the crack opening. In the current study, the initiation and propagation of induced crack are monitored via a high-speed camera.

2. EXPERIMENTAL PROCITURE

2.1. Testing material

In this study, a transparent material with low tensile strength and a rock-like behaviour is used. This transparent breakaway blend made from high-quality resin and polymers is found by experimentation. The material known as Smash-It (Barnes, 2017) has a glass-like clarity and shatter. In particular, the Smash-It material has been originally used on film and TV productions. To form specimens for physical models, Smash-It has to be first melted by heating it up somewhat prior to being hand-poured into moulds. The samples are the thick-walled cylinders with 100mm external and 8 mm internal diameters with a constant height of 15 mm. Further details regarding Smash-It sample preparation could be found in (Hamidi *et al.*, 2017(Under review)). Hamidi *et al.* (2017) tested the properties of Smash-It samples which are summarized in Table 1.

Table 1. Summary of the properties of Smash-It.

Density (gr/cm^3)	Porosity (%)	BTS (MPa)	UCS (MPa)	E (GPa)	ν (-)	K_{IC} (Mpa.m^0.5)
1.065	0	0.73	17.62	1.09	0.466	0.0498

BTS: Brazilian Tensile Strength
UCS: Uniaxial Compressive Strength
E: Elastic Modulus
ν: Poisson's Ratio
K_{IC}: Mode I fracture toughness

2.2. Experimental set-up

As mentioned, the ultimate goal of this research is to characterize crack initiation and propagation in two dimensions. The design aims for low-strength brittle materials that could reduce the complexity and even the cost of experimental set-ups where low-stiffness loading plates and connection tubes could be used instead. As a result, a two-dimensional (2D) set-up is designed to investigate the hydraulic fracturing. A schematic illustration of the main set-up is shown in Figure 3. The set-up has two simplified considerations. First, the samples are in a disc shape instead of a rectangular slab. This consideration will help to compare the results with a Lame solution for the hollow elastic cylinder. Second, the tests are conducted under atmospheric pressure (confining pressure equal to zero) instead of biaxial stress condition as it is usually applied in fracturing tests. Physical model tests of hydraulic fracturing have been conducted with different objectives, such as crack initiation (from either open or cased holes), crack propagation, crack interaction with discontinuities, and crack containment. With a few modifications, the current experimental set-up could be used for all the purposes.

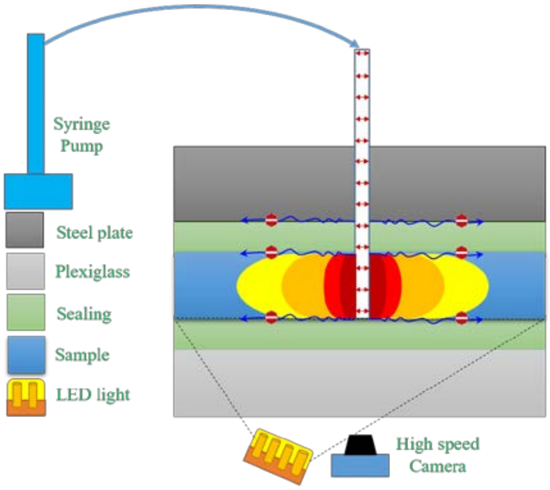

Figure 3. Illustration of the experimental set-up design (not to scale)

3. RESULTS

Testing is started by applying a constant flow rate (=1ml/min) via the syringe pump into the hole. Consequently, the fluid pressure inside the hole is increased up to a threshold in which the crack starts to initiate. Figure 4 presents the recording of fluid pressure and flow rate results obtained from a completed test. A sudden drop in fluid pressure graph is a representative of the rupture in the sample. The pressure at this point is the Breakdown Pressure (BP) of the material. The results (Figure 4) show that breakdown pressure under the zero confining stress is almost equal to the tensile strength (T) of the Smash-It material. After the first peak, the fluid is injected at a different flow rate to find the re-opening pressure of the induced crack. It can be seen by increasing the injection rate, the re-opening pressure increases as well.

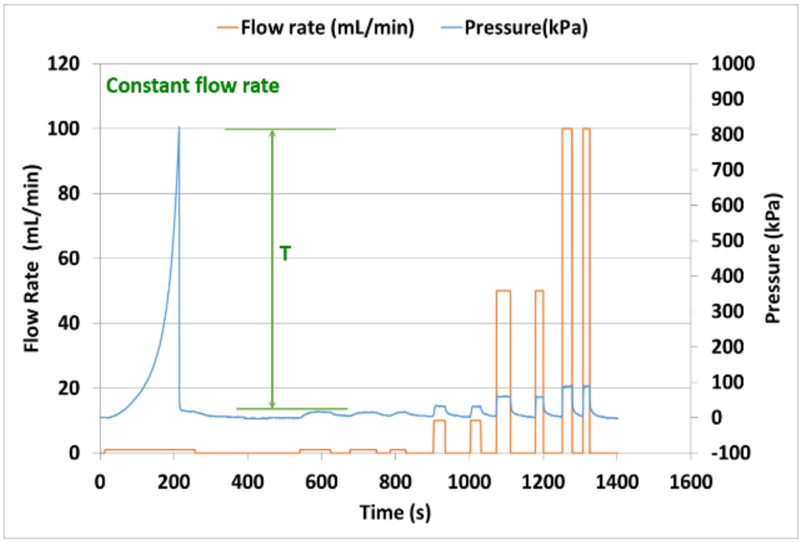

Figure 4. Illustration of flowrate-time and pressure-time record

4. CONCLUSIONS AND DISCUSSIONS

The aim of the present study is to assess the usefulness of a two-dimensional experimental setup for investigating crack initiation, propagation, interaction with discontinuities. An innovative experimental set-up is introduced for conducting hydraulic fracturing on a low-strength transparent rock-like material. The breakdown pressure for the low–strength synthetic material is measured and it is almost equal to the material tensile strength. Although such results have been obtained in former studies by conventional experimental set-ups, the presented set-up allows the investigation of materials with low-strength in a quasi 2D problem to investigate the influence of different physical parameters on fracture initiation and propagation in a simpler – and as such safer and cheaper – way.

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