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LECTURE 12

Generalized formulation of the bio heat transfer problem with regard to the soft tissue deformations-II

Lecture slides
for Bachelors of Technical Sciences

Санкт-Петербургский государственный политехнический университет
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Domains of Definition of Dispersion Dependences

- To construct diagrams of the dispersion relations we have to find values of x, at which the functions are real and positive. This function y(x) has three roots:

$$x_1 = (-1 - A + \sqrt{1 - 2A + A^2 + 4AA_4}\,)/2;$$

$$x_2 = (-1 - A - \sqrt{1 - 2A + A^2 + 4AA_4}\,)/2;$$

$$x_3 = -1 + A_4$$

- and a vertical asymptote

$$x_4 = -A - A_4.$$

Domains of Definition of Dispersion Dependences

- It is easy to show that $x_2<x_3<x_1$ and $x_2<x_4<x_1$ at all admissible values of constants “A”.
- At the same time, there are two possible locations of the roots x_3 and x_4:
  - if $A<1-2A_4$, then $x_3<x_4$ (**first case**);
  - if $A>1-2A_4$, then $x_4<x_3$ (**second case**).
- The radicand in y(x) takes positive values on two intervals, one of which contains the point x_1, and the other one contains x_2. These intervals correspond to different branches of the dispersion relation curves.

Horizontal Asymptotes

- The horizontal asymptotes are determined by the values:

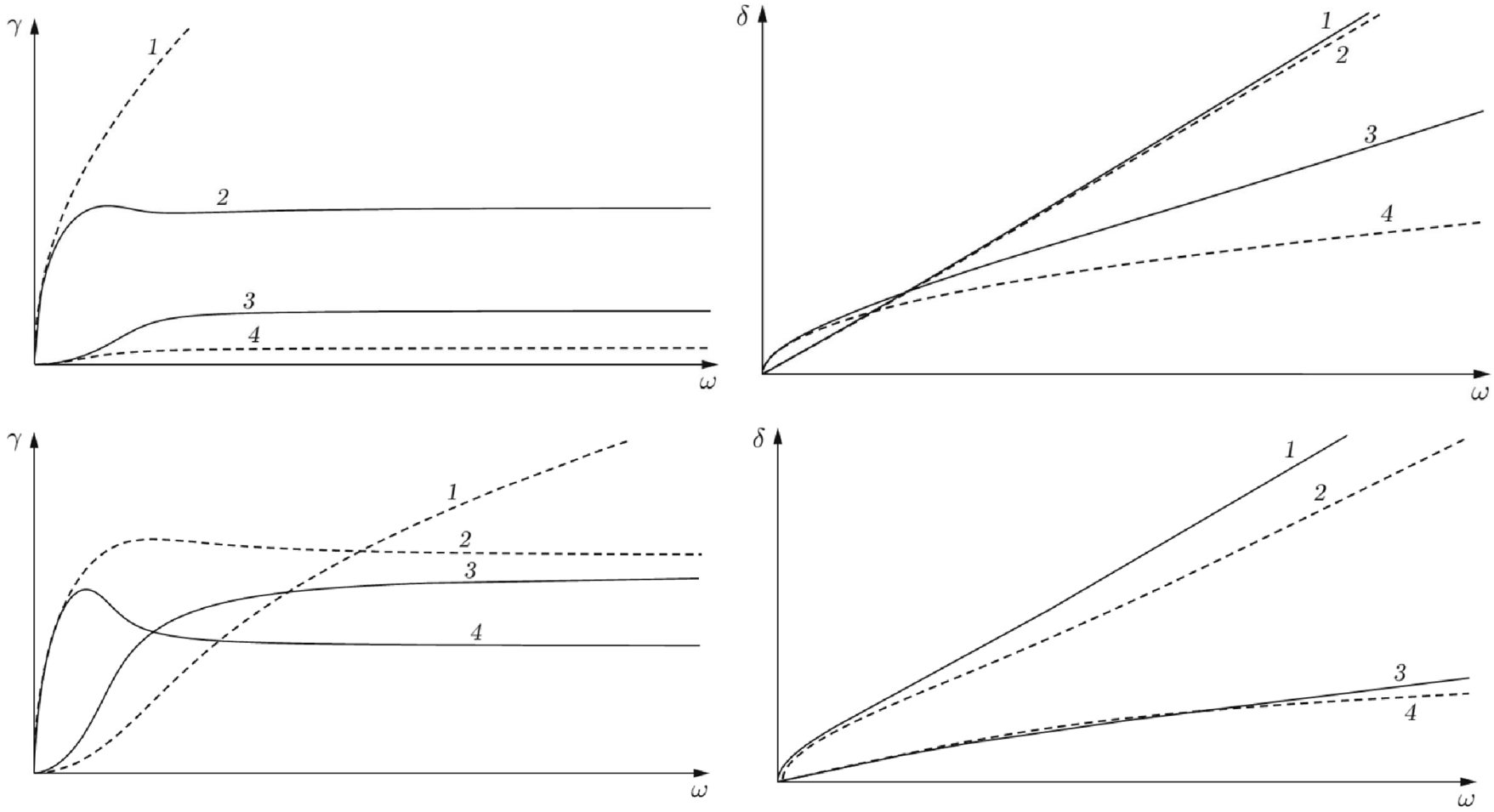
$$\gamma_1 = \frac{A_3(a-b)}{2a\sqrt{2(A_1+A_2-a)}}, \qquad \gamma_2 = \frac{A_3(a+b)}{2a\sqrt{2(A_1+A_2+a)}}.$$

- The expressions for the constants a and b have the form:

$$a = \sqrt{(A_1-A_2)^2+4A_1A_2A_4}, \qquad b = A_2 + A_1(2A_4-1).$$

- In the first case, the straight line $\gamma = \gamma_1$ is the asymptote of the thermal branches, and the straight line $\gamma = \gamma_2$ is the asymptote of the acoustic branches;

Dispersion curves



Oblique Asymptotes

- The diagram δ(ω) has only oblique asymptotes emanating from the origin:

$$\tan \varphi_1 = \sqrt{(A_1 + A_2 - a)/2};$$

$$\tan \varphi_2 = \sqrt{(A_1 + A_2 + a)/2},$$

- Where ϕ1 is the angle of the asymptote of the acoustic branches in the **second case** of the root locations and the thermal branches in the **first case** and ϕ2 is the angle of the asymptote of the thermal branches in the **second case** and the acoustic branches in the **first case.**

Group and phase velocity

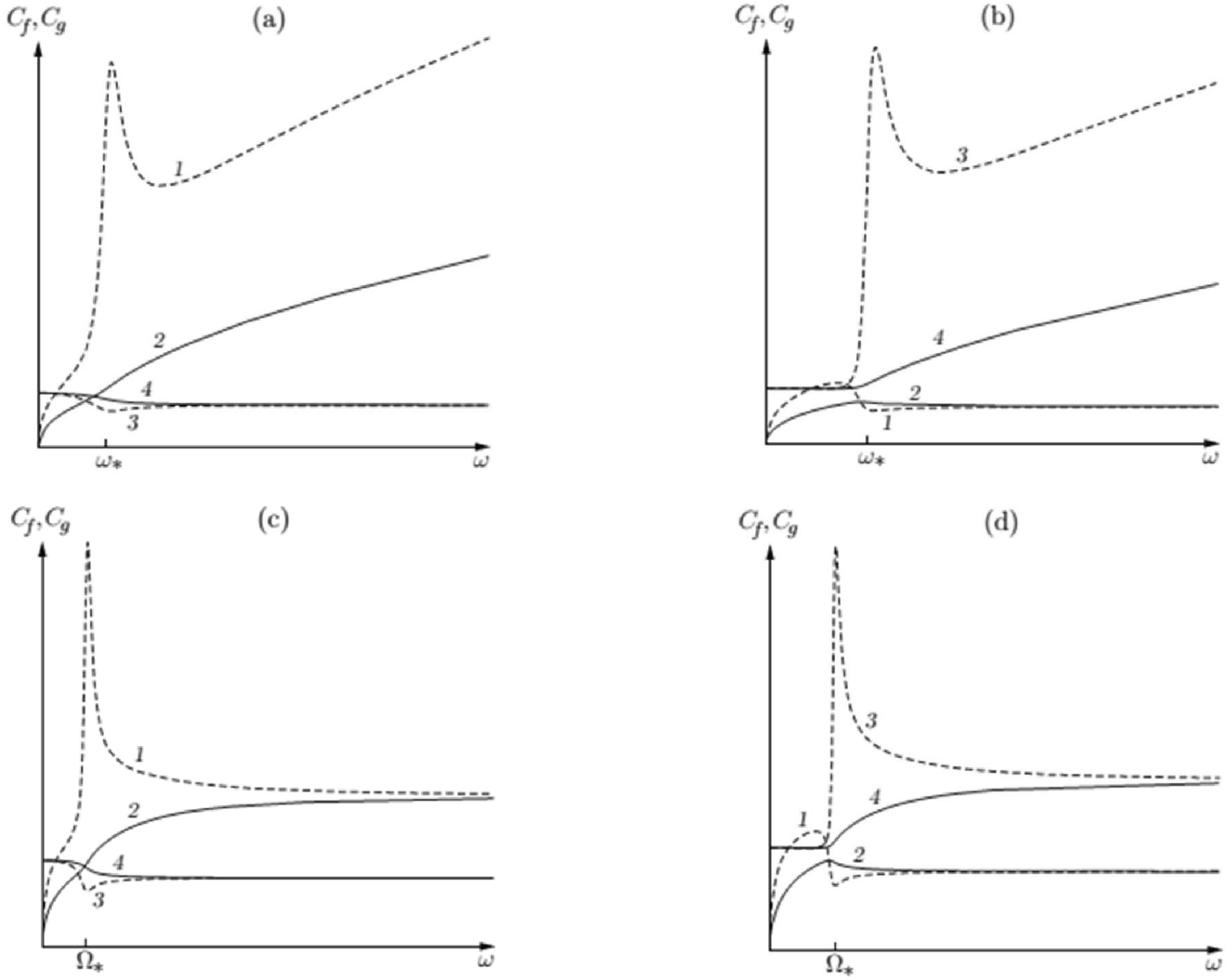
- We can obtain the dependence of the phase velocity C_f on the parameter x in the form:

$$C_f(x) = \frac{\omega(x)}{\delta(x)} = \frac{\sqrt{2}}{\sqrt{A_1\left(-x + \sqrt{x^2 + 4y^2}\right)}}.$$

At the same time the expression for the group velocity is too long to reproduce it here:

$$C_g(x) = \frac{d\omega}{d\delta} = \frac{d\omega}{dx}\left(\frac{d\delta}{dx}\right)^{-1}$$

Group and phase velocity



The critical constant of the heat-flux relaxation

- In **first case**, we have $\tau < \tau_0$, and in **second case**, we have $\tau > \tau_0$. Here τ_0 is the critical constant of the heat-flux relaxation:

$$\tau_0 = \frac{\lambda\rho(\rho c_v(K + 4G/3) - \alpha^2 K^2 T_*)}{(\rho c_v(K + 4G/3) + \alpha^2 K^2 T_*)^2}.$$

- From the formula it follows that the critical relaxation time of the heat flux is directly proportional to the thermal conductivity of the material.
- The critical relaxation times for real materials correspond to the estimate τ_0.