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Rough contacting surfaces with elevated contact areas

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ABSTRACT

We consider two elastic half-spaces contacting along multiple contact spots. In order to provide contact, these spots should be somewhat elevated; moreover, they may take the form of "columns" that connect the half-spaces. The effect of this factor on the overall compliances (normal and shear) is examined in 3-D setting, as function of the shape of the "columns". This effect is found to be strong, and it is quite sensitive to the mentioned shape. The extent of coupling between the shape of the "columns" and interactions between them, and of the elastic contrast between the "columns" and the main material are also examined.

1. Introduction

Rough contacting surfaces have been considered in literature, both in the context of conductivity and elasticity, since at least 1950′s; we mention several publications that are relevant to the present work. In the context of conductivity, we note the key work of Greenwood (1966) on conductance across multiple contacts and the effect of interaction between contacts. In the context of elasticity, focusing on the linear formulation (incremental compliances of contacts) (low applied stresses, resulting in negligible change of contact areas), we mention the work of Greenwood and Williamson (1966) and the review of earlier works given in the paper of Ciavarella, Demelio, Barber, and Yang (2000). In the work of Barber (2003), the conductivity-normal incremental compliance connection was established for two half-spaces contacting along arbitrary arrangement of contact spots of arbitrary geometries; it was applied by Sevostianov and Kachanov (2009) to reformulation of the results of Greenwood (1966) on conductance to *elastic* interactions of contacts; they also extended the cross-property connection to shear loading; the extension of the cross-property connection to transversely isotropic half-spaces was given by Kuzkin and Kachanov (2015). We also mention somewhat related work of Kachanov, Prioul, and Jocker (2010) on similarities and differences between rough fractures with contacting surfaces and traction-free cracks.

In discussing the effect of contacts, the contact areas are usually treated simply as flat spots along the interface between half-spaces. However, in order to provide contact, these spots should be somewhat elevated; they may even have "columnar" shapes (see Laubach, 2003, Fig. 1). This factor does not seem to have been sufficiently analyzed in literature. Such configuration was considered by Sayers, Taleghani, and Adachi (2009) from the viewpoint of its effect on the overall elastic properties (and hence on wavespeeds) – by computing the average displacement discontinuity across an interface

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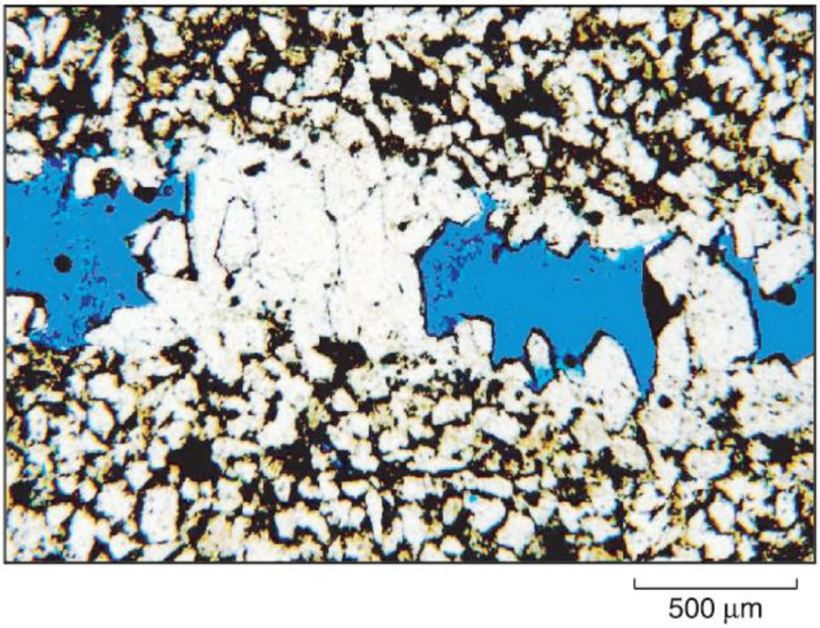

Fig. 1. Contacts with columnar shapes in quartz. Image was made with scanning electron microscope (after Laubach, 2003).

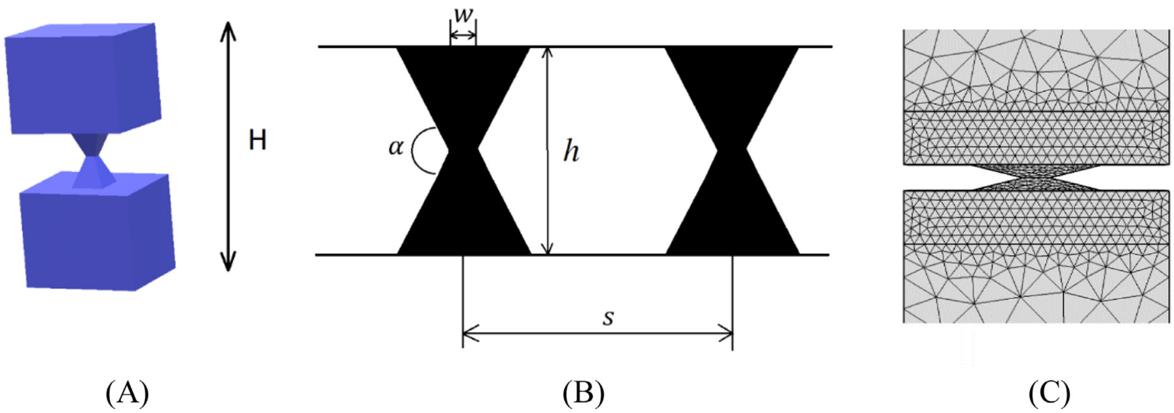

Fig. 2. The geometry of the configuration (the periodic cell, (A), the array of contacts, (B), and the finite element mesh, (C)).

with "columnar" contacts; the limitation of this work was a 2-D formulation and the assumption that the connecting 2-D "columns" had the shape of rectangles of constant width.

The present work considers elevated contacts in 3-D formulation. It focuses, primarily, on the effect of *shapes* of connecting columns. We also discuss the extent of coupling between the shapes of columns and interactions between them. The problem is discussed in the context of effective stiffnesses (normal and shear) of the contacting surfaces. The analysis is done numerically, by FEM simulations.

2. Analysis

The stiffnesses of the considered configuration strongly depend on the statistics of mutual positions of contacts (as discussed in the above-mentioned works of Sevostianov and Kachanov (2009) and Kachanov et al. (2010)). In order to exclude this factor and focus on the effect of contact shapes, we consider a periodic system of columns.

Formulation of the problem. Thus, we consider a 3-D doubly-periodic system of columns that connect two half-spaces; the configuration of one periodic cell is shown in Fig. 2(A)–(B). The shape of columns is prismatic (square cross-section) and is controlled by angle α that can be viewed as shape factor. Parameter s/w controls spacing between contacts (and hence the strength of contact interactions).

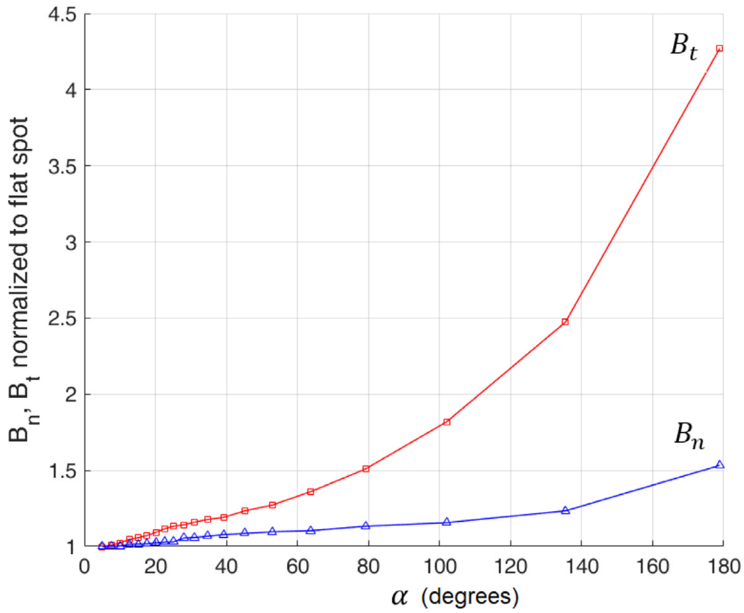

Fig. 3. Values of B_n and B_t, normalized to their values for flat contact, as functions of angle α ($s/w = 25$, $\gamma = 1$, $\nu = 0.25$).

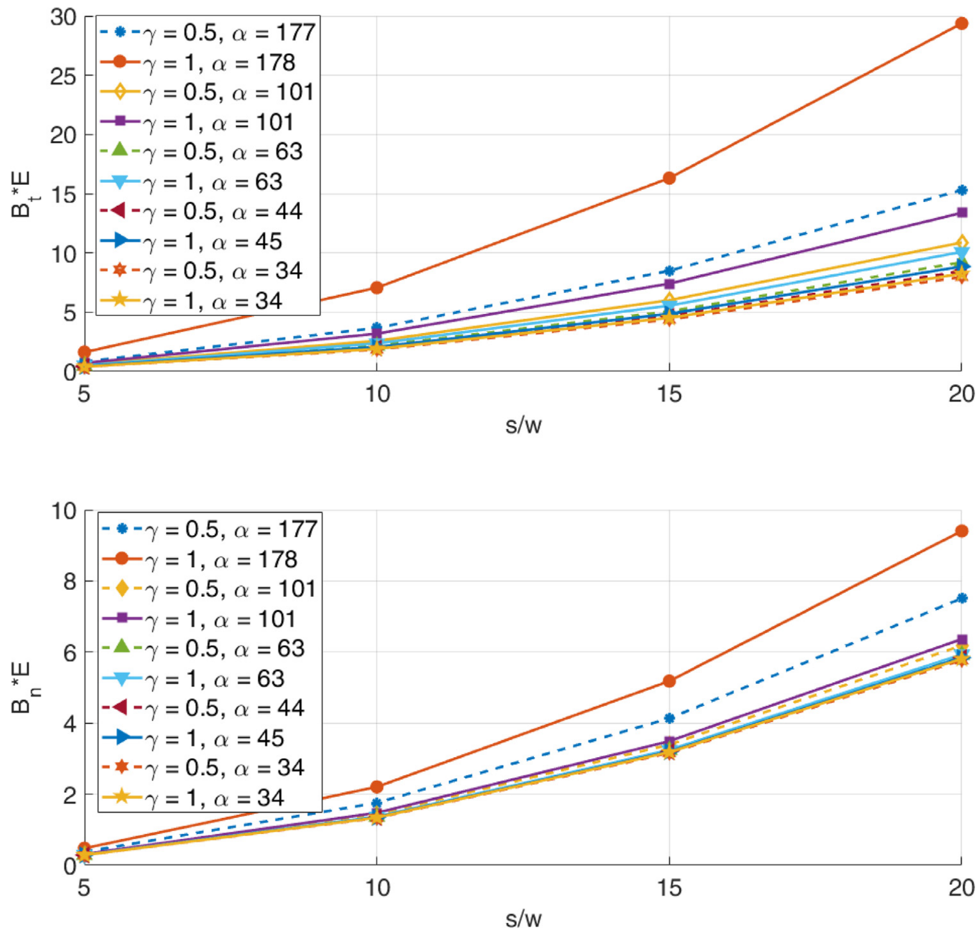

Fig. 4. The dependence of B_t and B_n on distances between the contacts, at several values of, assuming $\gamma = 0.5$ (dashed line) and $\gamma = 1$ (solid line).

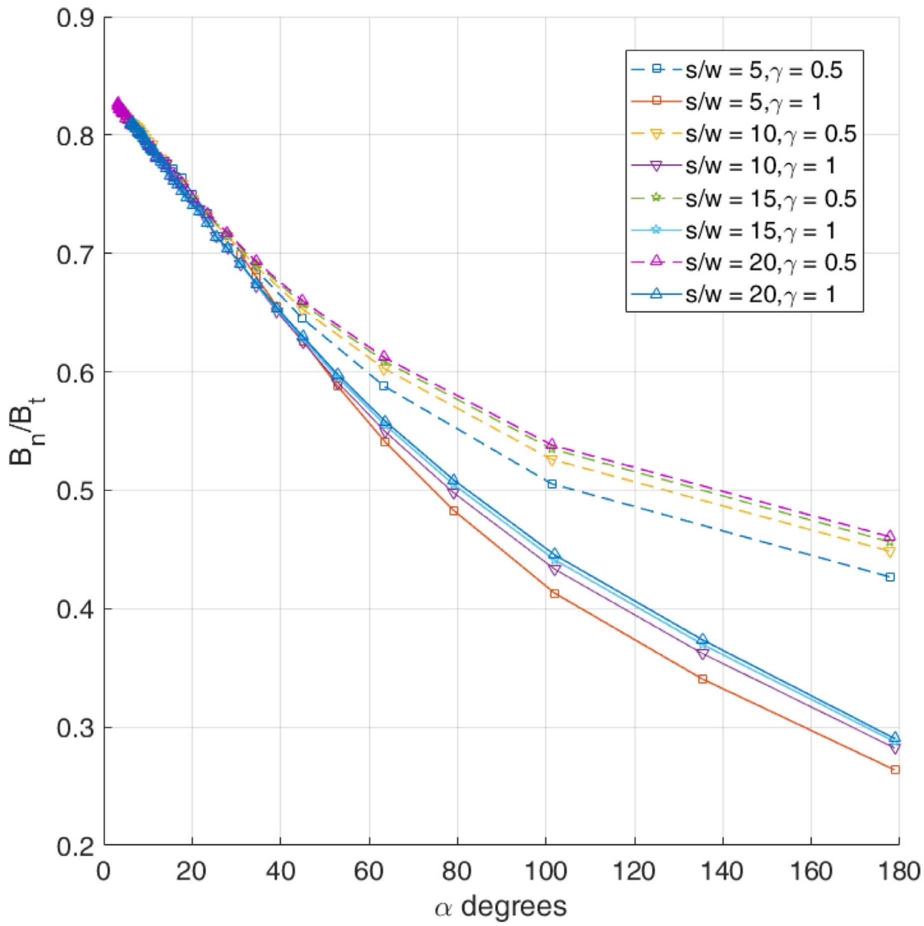

Fig. 5. The dependence of B_n/B_t on angle α at different values γ and s/w. Dashed lines correspond to $\gamma = 1$, solid lines to $\gamma = 0.5$. ($\nu = 0.25$).

The dimensionless height of a prism h/w is an independent geometric parameter if $\alpha > \alpha_{\min}$ where $\alpha_{\min}$ is the value of α at which the contacting surfaces touch one another; otherwise, h/w is expressed in terms of s/w and α (Fig. 2):

$$\frac{h}{w} = \begin{cases} \gamma, & if\ \alpha \geq \alpha_{min} \\ \frac{(s-w)}{w} tan\frac{\alpha}{2}, if & \alpha < \alpha_{min} \end{cases}, \tag{2.1}$$

where

$$\alpha_{min} = 2\arctan\left(\frac{\gamma w}{s-w}\right) \tag{2.2}$$

Remark. The square shape of contacts (implied by the prismatic shape of the columns) ensures isotropy in the tangential directions. Indeed, the relation between the traction vector $\boldsymbol{t}$ transmitted through the contact and the resulting average vector of the relative displacement of the upper and lower half-spaces $\mathbf{b} \equiv <[\mathbf{u}]>$ is given by

$$b_i = B_{ij} t_j \tag{2.3}$$

where $\boldsymbol{B}$, being a symmetric (as follows from the reciprocity theorem) second-rank tensor, does not distinguish, from the symmetry viewpoint, between a square and a circle.

In the FEM simulations, periodic boundary conditions for displacements in the x- and y-directions were imposed. Traction vector that corresponds to the applied normal and shear loads is prescribed at the upper and lower faces of the periodic cell. Calculations for the one-cell problem covered the range of column shapes $0 < \alpha < \pi$; the extreme cases $\alpha = 0$ and $\alpha = \pi$ correspond to absence of the elevation (flat contact spots) and to columns of constant thickness, respectively. The dependence on angle α was examined for several values of parameter s/w. Two loadings, normal and shear, were applied and the average displacement discontinuity across stress free parts of the surfaces were calculated. The COMSOL 5.3 package was used, with denser mesh near crack faces and contacts, as shown in Fig. 2(C). The calculations were aimed at finding the normal and shear compliances B_n,B_t and their ratio (that is of importance in geomechanics applications). The compliances form the tensor $\boldsymbol{B} = B_n\boldsymbol{nn} + B_t(\boldsymbol{I} - \boldsymbol{nn})$ that enters formula (2.3); $\boldsymbol{I}$ is the second-rank unit tensor.

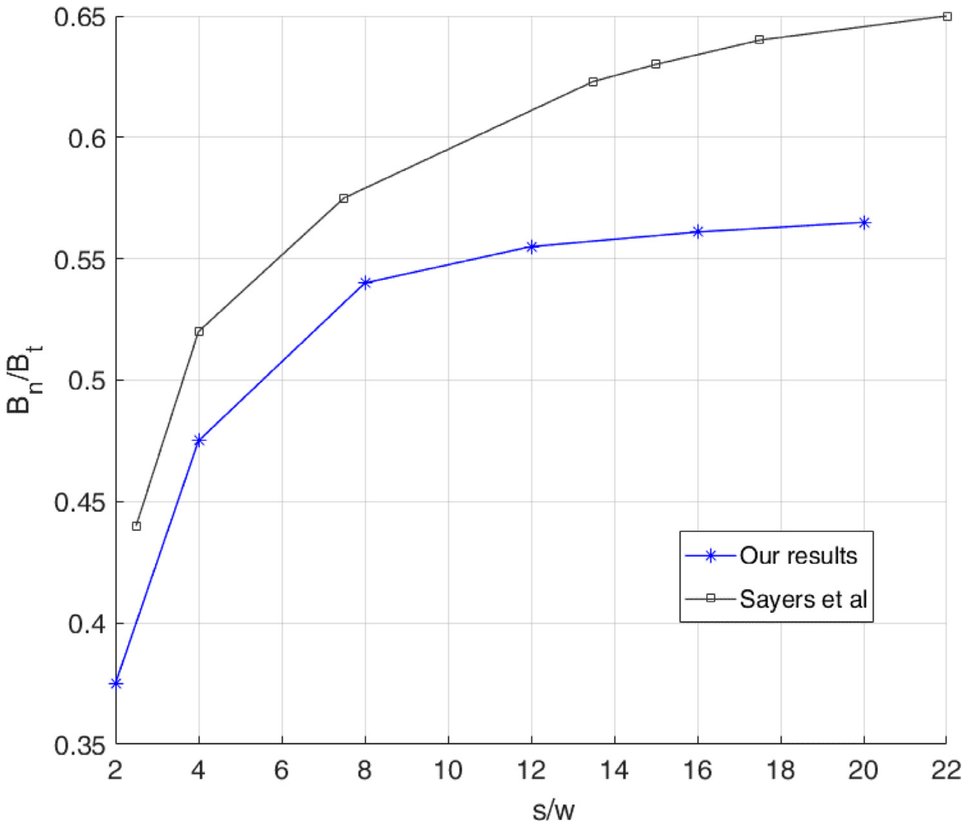

Fig. 6. Comparison of the obtained results (at $\boldsymbol{\alpha} = \boldsymbol{\pi}$) for the ratio B_n/B_t with 2-D results of Sayers et al. (2009).

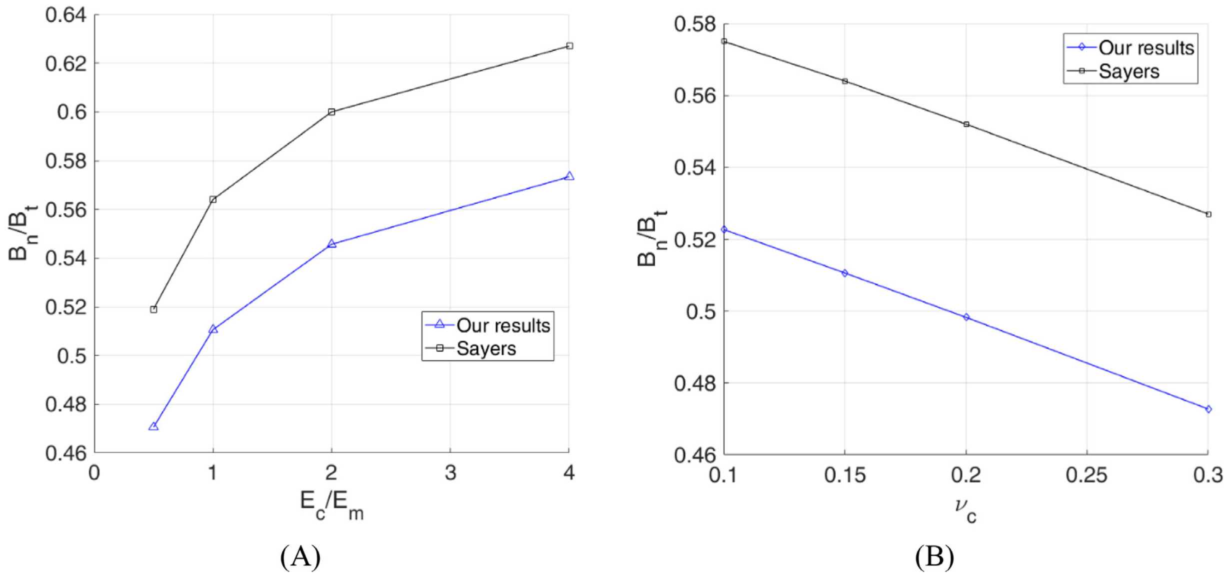

Fig. 7. The effect of contrast in Young's modulus (A) and Poisson's ratio (B) between the columns and the main material on the ratio B_n/B_t, for columns of constant thickness ($\alpha = \pi$). Comparison with results of Sayers et al. (2009) (where the same values of elastic constants were assumed) is shown.

The "height", H, of the periodic cell (Fig. 2(A)) should be sufficiently large, in order to adequately simulate semi-infinite half spaces. In the simulations, we examined the convergence of the ratio of the normal and shear compliances, B_n/B_t, as H increases. The calculations showed that the value $H/w = 25$ is sufficient for simulation of half-spaces.

Normal and shear compliances and their ratios, as functions of column shapes. The values of the said compliances, normalized to their values for flat contacts (no elevation), as functions of angle α are shown in Figs. 3 and 4. It is seen that both B_t and B_n increase with α, with B_n being much less sensitive to α. This difference shows that the contacts *cannot* be replaced by some "equivalent" columns of constant thickness – hence emphasizing that shape factor (angle α) cannot be ignored. Thus, the shape factor has strong effect on the compliances. In particular, it significantly affects the value of B_n/B_t that controls the ratio of the normal and shear wavespeeds – the quantity of interest in geophysics.

Fig. 6 shows the dependence of the ratio B_n/B_t on angle α (shape factor) at different values of s/w and γ. It is seen that the effect of α is quite substantial, whereas the effect of s/w (that characterizes the strength of the interaction effect) is relative weak. Thus the two effects – of contact shapes and of contact interactions – are only weakly coupled (at least, at spacings $s/w > 10$). Fig. 3. Values of B_n and B_t, normalized to their values for flat contact, as functions of angle α ($s/w = 25, \quad \gamma = 1 \;, \nu = 0.25$).

Fig. 6 compares our results with the ones obtained by Sayers et al. (2009) in the framework of 2-D model involving rectangular columns of constant width. Their 2-D configuration corresponds, in our 3-D formulation, to the case $\alpha = \pi$ (columns of constant width). Noticeable difference is seen between the two models. This illustrates the fact (well known in the context of cracks, Kachanov, 1994) – that one has to be careful with replacing 3-D configurations by 2-D models.

The effect of elastic contrast between the columns and the half-spaces. We now assume that Young's moduli of the columns (E_c) and of the main material (E_m) are different. In order to make comparison with results of Sayers et al. (2009), we assume that columns have constant width ($\alpha = \pi$). Also, we assume that $\gamma = 0.44$ and $\frac{s}{w} = 7.5$. Fig. 7 shows the calculated values of the ratio B_n/B_t. It is seen that the effect of elastic contrast is relatively mild (qualitatively, similar to the predictions of Sayers et al., in spite of some quantitative differences).

3. Discussion and conclusions

We considered two half-spaces contacting along elevated contact spots having columnar shapes. We examined the effect of columns shapes (and some other parameters) on the normal, B_n, and shear, B_t, compliances of the interface, and on their ratio B_n/B_t (that is of particular interest in geophysics applications). The main findings are:

- The effect of column shape, as characterized by angle α, is quite strong and it is different for B_n and B_t - implying that columns cannot be replaced by some "equivalent" columns of constant thickness (Figs. 3 and 5);
- The effects of contact shape and of the interaction between contacts (as characterized by spacings s/w) seem to be only weakly coupled, at least at spacings $s/w > 10$);
- Comparison with 2-D results of Sayers et al. (2009) – where contact shapes were assumed rectangular shows similar qualitive trends but noticeable quantitative differences, as far as the normal and shear compliances, and their ratio B_n/B_t are concerned.

Numerical simulations have been carried out using computational facilities of the Supercomputer Center "Polytechnic" at Peter-the-Great Saint Petersburg Polytechnic University.

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