Brittle fracture of rocks under oblique impact loading

I. Asonov I. Berinskiy J. Ing A. Krivtsov S. Le-Zakharov E. Pavlovskaia
M. Wiercigroch
iberinsk@gmail.com

Abstract

In this paper the effects of oblique impact loading of brittle rocks are investigated. The drilling process in hard rocks is simulated using particles dynamics (PD). The rock sample and impactor are described by particles with different bond strength. Impact is generated by applying a dynamical force to the impactor. The results are compared with the predictions obtained according to a simple analytical model of a drifting oscillator [1]. It is shown that the models correspond to each other reasonably well but there are a number of differences in their predictions. The results of the computer simulations aim to provide a valuable insight into the brittle fracture of rocks subjected to impact loading and eventually improve analytical model of this process.

1 Introduction

This paper is devoted to the modelling of brittle fracture in rocks subjected to oblique impact loading. For this purpose a numerical model using particle dynamics (PD) technique was developed, which is based on the earlier model by Krivtsov *et al.* [2]. In new numerical model the type of loading was changed for better simulation of real drilling process. To validate the obtained results, the PD simulations are compared with predictions using the analytical model which was proposed in [1]. This simplest theoretical model includes a massive rigid body (the drilling tool) subjected to static and dynamic excitation forces and interacting with the drilled formation, which was model by the constitutive equations of the material resistance based on dry friction. In the case of a one dimensional representation this can be reduced to a system with one degree of freedom, but due to essential nonlinearity even this elementary description leads the equations which can be solved analytically only in some certain cases. Despite quite radical simplifying assumptions, the model was able do describe the fall of material removal rate for a higher static loading with a good agreement to experimental investigations. In [1] the material removal rate was investigated as a function of the static force and the amplitude of the harmonic force. Later in [3]–[4] a more complex model was proposed where the visco-elastic properties of the drilled formation have been taken into account. As a result the system moves forward in stick-slip phases, and its behavior may vary from periodic to chaotic motion. In the current work numerical PD simulations are used to consider a number of new features such as rotation torque of the tool, explicit debris removal, and geometry of the contacting bodies.

2 Particle Dynamics model

In particle dynamics method the objects (tools, targets etc.) are made of particles arranged in crystal lattices. Every particle represent some part of the object material such as a grain or a domain. Usually 10^3—10^4 particles are used for the standard tests and 10^5—10^6 particles are used to obtain more accurate results. In this work two massive bodies are considered, namely the tool (drill bit) and the specimen (rock material). A 2D model with a close-packed hexagonal lattice is constructed and the motion of the particles is obtained through integration of the Newtonian equations of motion with the prescribed interaction forces:

$$F_1(r) = \begin{cases} F(r) & \text{for } 0 < r \le b; \\ k(r)F(r) & \text{for } b < r \le a_{cut}, \end{cases} \tag{1}$$

where $F(r)$ is the force corresponding to the Lennard-Jones potential with the equilibrium distance a and bond energy D; quantity b is the brake distance for the Lennard-Jones interaction:

$$F(r) = f_0 \left[\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^{7} \right], \qquad f_0 = \frac{12D}{a}, \qquad b = \sqrt[6]{\frac{13}{7}} \approx 1.1a; \tag{2}$$

$k(r)$ is a shape function:

$$k(r) = (1+\alpha)\left(1 - \left(1 + \sqrt{\frac{\alpha}{1+\alpha}}\right)\left(\frac{r^2 - b^2}{{a_{cut}}^2 - b^2}\right)^2\right)^2 - \alpha\,, \tag{3}$$

where α is a positive parameter, defining brittleness of the material, $a_{cut} > b$ is the cut-off distance for the interaction. Increasing α results in the repulsion appearing in the vicinity of the cut-off distance. This produces a potential barrier for joining two particles, which were separated due to the impact fracture. The parameter α changes from 0 to 2, and the value $\alpha = 2$ corresponds to a brittle material. The geometry and loading of the drilling tool is shown at Fig. 1.

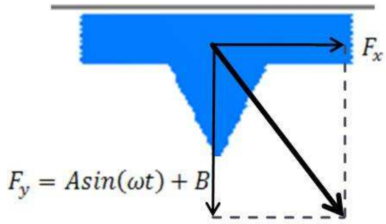

Figure 1: Geometry and loading applied to the drilling tool

Periodic kinematic boundary conditions are applied on the left side and on the right side of the tool and the specimen, and the specimen is rigidly fixed at the bottom. The material of the drill bit is considered to be 10 times harder than material of the specimen. It is simulated with different bond energies for the particles of the specimen and the drill bit.

The applied loading in the new model is closer to reality than in the previous one [2], where the initial velocities of the particles were applied before every strike of the drill bit. Instead, now the dynamical conditions are applied to every particle at every time step. Thus the external axial force $F_y = A \sin \omega t + B$ directly corresponds to the force used in the analytical model [1]. The shear loading force F_x is introduced to model a rotation torque. As a result the external loading is described by three independent parameters namely by the amplitude of dynamic axial force A, magnitude of static force B, and the shear force F_x (Fig. 1).

Parameter	*Value*
Particles number	$(5–30)\cdot 10^3$
Avg. thermal velocity	$0.01v_d$
Number of strikes	5–10
Time of modeling	$(147–295)T_0$
Period of strikes	$(4.69–100)T_0$
Integration step	$0.02T_0$
Cut-off radius (a_{cut})	$1.4a$
Brittleness (α)	2
Strength: Tool/Specimen (D_1/D_2)	10

Table 1: Parameters of the model.

Numerical parameters of the model are presented in table 1. Here v_d is a dissociation velocity of the particle; T_0 is a period corresponding to the natural frequency of the particle as

$$T_0 = 2\pi\sqrt{\frac{C}{m}}, \qquad v_d = \sqrt{\frac{2D}{m}}, \tag{4}$$

where m is the particle mass and $C = -F'(r) = 72D/a^2$ is the stiffness of particle interaction.

It should be noted here that the proposed numerical model also allows to simulate wear and tear of drill bit as well as the rock fracture.

3 Results of the particle dynamics simulation

A numerical study was conducted to identify the optimal parameters of loading. Static tests for model with about 10000 particles indicate that a value of $0.01f_0$ may be chosen as a maximum value of static loading force B. The value of $B = 0.002f_0$ was chosen as a minimum value. The simulations were carried out for a number of B values taken with the step of $0.002f_0$. It was obtained in [1] that the optimal ratio between the static force and the amplitude of the dynamic force is $B/A = 0.387$ and therefore this value was used in computer simulations. For the chosen values of axial loading the dependence between the shear force F_x and material removal rate (MRR) was constructed and the results are shown in Fig. 2. MRR is defined here as the quantity of material removed during one strike of the tool, and the average MRR for different distributions of the initial velocities of the particles (different heat distribution) is presented. It is assumed that the material is removed if it has been divided into separate particles. To make comparison of the obtained results easier, a scaled value of MRR has been introduced as

$$\widehat{MRR} = \frac{MRR - MRR_{min}}{MRR_{max} - MRR_{min}}, \tag{5}$$

where MRR_{min} and MRR_{max} are the minimum and maximum value of MRR for given B.

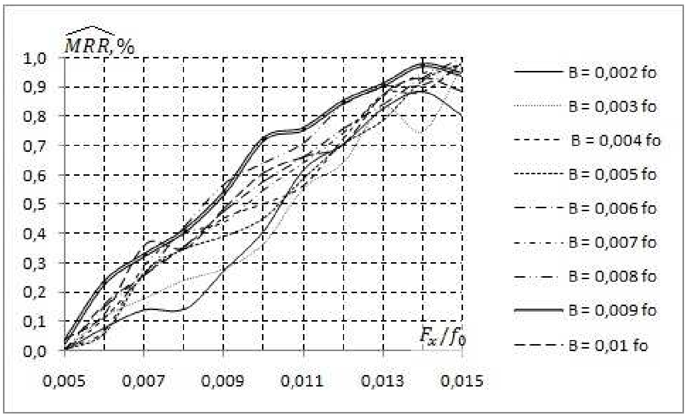

Figure 2: $\widehat{MRR}$ as a function of the shear force F_x computed for different values of the static force B.

An average $\widehat{MRR}$ as a function of the shear force F_x is shown for different static force values B in Fig. 2. As can be seen from these graphs, for all values of the static force the modified material removal rate shows the same trend. It grows with the increase of the shear force. In order to obtain the results independent of the static force value, averaging of $\widehat{MRR}$ over static force B for each value of the shear force has been done, and the result is shown in Fig. 3. As one can see the obtained dependence is near the linear. Therefore the difference of $\widehat{MRR}$ and a linear fit (LF) versus the shear force F_x has been calculated and is presented in Fig. 4. From the results in Fig. 4 the optimal value of F_x where the removal rate is maximal is obtained as $0.007f_0$. It should be noted that as simulations have shown values of F_x bigger than $0.01f_0$ could lead to fast wear and destruction of the tool.

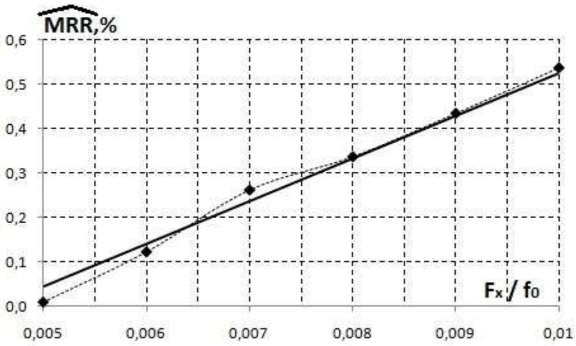

Figure 3: $\widehat{MRR}$ averaged with respect to B as a function of the shear force F_x.

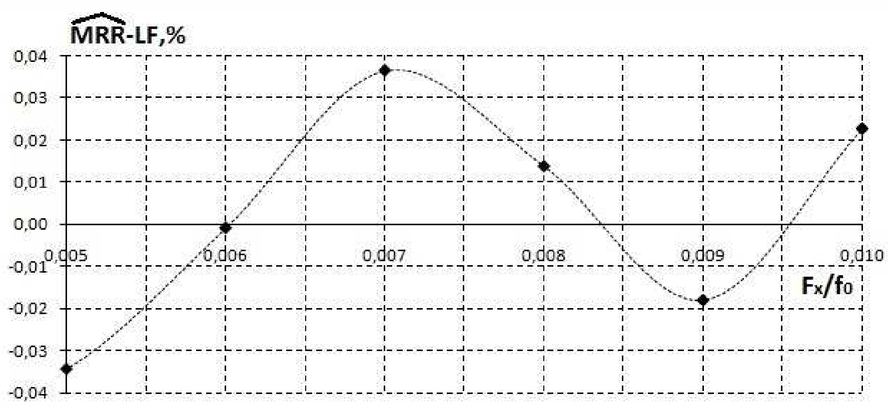

Figure 4: $\widehat{MRR}$ - LF as a function of the shear force F_x.

Further a value of $F_x = 0.007f_0$ is chosen to plot the diagrams for $\widehat{MRR}$ dependencies on B/A and A/B (Fig. 5 – Fig. 6). Here R_{max} is a maximum value of $\widehat{MRR}$ for the minimal of considered A.

4 Analytical dry friction model

Let us compare results of computer simulations obtained above with the results of analytical modeling proposed in[1]. Below for convenience of reader the equations of motion and some results are included. In this model mass m is used to represent the tool, $F(t)$ is the overall drilling force, $P(\dot{y})$ is the resistive force, x is the displacement of the tool's tip, and y is the displacement of the dry friction element, which represents the progression of the drilling surface. The equation of motion of the mass takes the following form

$$x < y \Rightarrow m\ddot{x} = F(t)\,, \qquad x > y \Rightarrow m\ddot{x} = F(t) - P(\dot{y}). \tag{6}$$

which depends on the relative displacement between x and y. In turn, the equation of motion for the slider can be expressed as

$$\dot{x} \geq 0 \Rightarrow y = x\,, \qquad \dot{x} < 0 \Rightarrow y = 0. \tag{7}$$

For simplicity is assumed that the overall drilling force $F(t)$ has the form

$$F(t) = A \sin \Omega(t - t_0) + B\,, \tag{8}$$

where A and Ω are the amplitude and frequency of the harmonic force, B is the static force, t the time, and t_0 is the time constant. The resistive force $P(\dot{y})$ is modelled usig Coulomb dry friction

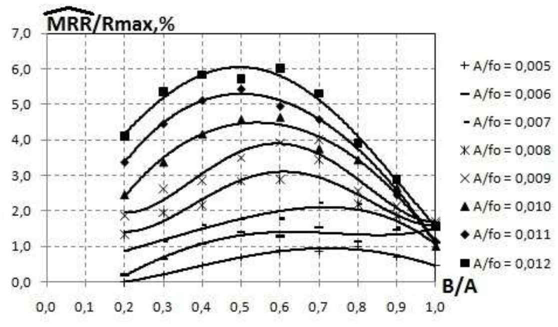

Figure 5: MRR as a function of the relative force B/A (A = constant) for different A/f_0

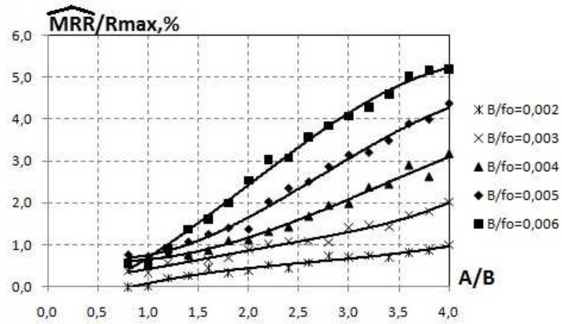

Figure 6: MRR as functions of the relative amplitude of the harmonic force A/B (B = constant) for different B/f_0

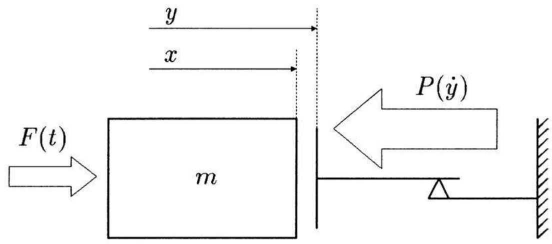

Figure 7: Dry friction model. Adopted from [1].

fulfilling the following conditions

$$\dot{y} > 0 \Rightarrow P(\dot{y}) = Q\,, \qquad \dot{y} = 0 \Rightarrow P(\dot{y}) \leq Q\,, \tag{9}$$

where Q stands for the modulus of the dry friction force. It is worth noting that the resistive coefficient, Q has a distinct value for any drilled material. On the contrary, the amplitude of the harmonic force and the static force can vary, and can be used as control parameters for the drilling process.

MRR as a function of the relative force, B/A (A = constant) and the relative amplitude of the harmonic force, A/B (B = constant) from [1] is shown in Fig. 8 – 9. Different graphs correspond to varying values of the relative excitation amplitude, $a = A/Q$ for Fig. 8, and to different values of the relative force, $b = B/Q$ as depicted in Fig. 9. The lowest (thick) curve in both figures corresponds to the small excitation case $a \to 0$ or $b \to 0$. In Fig. 8 higher curves are calculated for $a = 0.1;\ 0.2;\ \ldots;\ 0.5$. In Fig. 9 higher curves correspond to $b = 0.05;\ 0.10;\ \ldots;\ 0.25$. Both figures are related to the maximum values of the MRR for the small excitation (R_{max}) that depend on friction force Q [1].

The following comparison of the functions of MRR obtained from different models can be made. From Fig. 8 it follows that the MRR function of the static force, B (while A is kept constant) has a well-pronounced maximum, which is taken at $B = 0.39A$ for a small excitation and shifts to the right for greater excitation. Looking at Fig. 5 it can be noticed that the MRR functions has maxima near $B = 0.7A$ but there is no exact dependence of the maximum displacement on different B. From Fig. 9 it follows that the MRR is monotonically increasing function of the amplitude of the harmonic force, A. In the case of small excitation this function becomes constant. Looking at Fig. 5 one can notice the monotonic dependence but there is no asymptotic limit. The difference between the results may be connected with the different behavior of the material in the considered models. For example, the drill bit penetrates into the specimen for any dynamical load in analytical model. Instead, the numerical model has a lower limit of loading so that if the loading is smaller than this there is purely elastic deformation of the rock. On the other hand the R_{max} values for

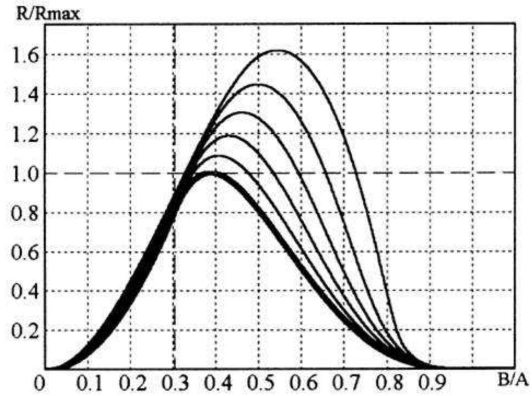

Figure 8: MRR as functions of the relative force B/A (A = constant) for vatying A/Q. Adopted from [1].

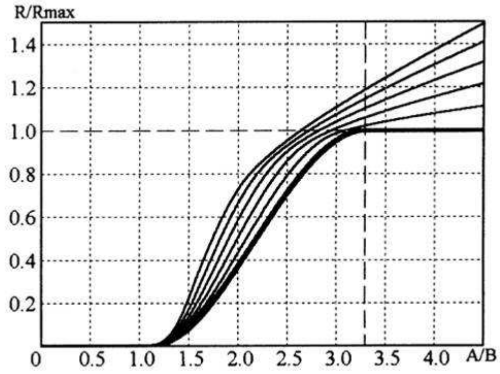

Figure 9: MRR as functions of the relative amplitude of the harmonic force A/B (B = constant) for varying B/Q. Adopted from [1].

the analytical model differ from that values for the numerical model because it is hard to determine the explicit analog of the dry friction force Q in the PD model. In further work the comparison of the numerical model with more complex analytical model considered in [3]–[4] will be undertaken.

5 Concluding Remarks

In this work drilling process of hard rocks was simulated using particle dynamics method. As a result new numerical model based on particle dynamics method was developed. The main difference of the presented model with the previous one is dynamical loading. This type of loading is more realistic and it is better for comparison with the analytical model. The comparison shows that the numerical model could give similar results to the results obtained with the analytical model. A value of the optimal relation between the static and dynamical amplitudes of the axial force was found. It differs from the previous result obtained with the analytical model. The optimal value of the shear force F_x was found as well. In further work the numerical model must be validated with an experimental data to obtain the qualitative results.

Acknowledgements

The support from RFBR (grant 09-01-92603-KO_a) and the Royal Society of London is gratefully acknowledged.

References

[1] A. M. Krivtsov, M. Wiercigroch. Penetration rate prediction for percussive drilling via dry friction model. Chaos, Solitons and Fractals 11 (2000) 2479–2485

[2] A. M. Krivtsov, E. E. Pavlovskaia, M. Wiercigroch. Impact fracture of rock materials due to percussive drilling action. XXI ICTAM, 15–21 August 2004, Warsaw, Poland

[3] E. Pavlovskaia, M. Wiercigroch. Modeling of an impact system with a drift. Physical Review E 64 (2001) 056224

[4] E. Pavlovskaia, M. Wiercigroch. Periodic solution finder for an impact oscillator with a drift. Journal of Sound and Vibration 267 (2003) 893–911

Igor Asonov, St. Petersburg State Polytechnical University, Russia

Igor Berinskiy, Institute for Problems in Mechanical Engineering of Russian Academy of Sciences, Russia
James Ing, University of Aberdeeen, United Kingom
Anton Krivtsov, Institute for Problems in Mechanical Engineering of Russian Academy of Sciences, Russia
Sergey Le-zakharov, St. Petersburg State Polytechnical University, Russia
Ekaterina Pavlovskaia, University of Aberdeeen, United Kingom
Marian Wiercigroch, University of Aberdeeen, United Kingom