Федеральное государственное бюджетное образовательное учреждение высшего профессионального образования
САНКТ-ПЕТЕРБУРГСКИЙ ГОСУДАРСТВЕННЫЙ ПОЛИТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ

НАЗВАНИЕ ФАКУЛЬТЕТА ИПММ

НАВАНИЕ КАФЕДРЫ Теоретическая механика

Elena Vilchevskaya

Lecture 4

Tensor analysis

Saint Petersburg State Polytechnic University
2014

Orthogonal tensors

A tensor **Q** of order two is said to be *orthogonal* if it satisfies the equality

$$\mathbf{Q} \cdot \mathbf{Q}^T = \mathbf{Q}^T \cdot \mathbf{Q} = \mathbf{E}.$$

We see that $\mathbf{Q}^T = \mathbf{Q}^{-1}$ for an orthogonal tensor

Furthermore, $\det \mathbf{Q} = \pm 1.$

We call **Q** a *proper* orthogonal tensor if det**Q** = +1; we call **Q** an *improper* orthogonal tensor if det**Q** = −1.

The operator defined by the orthogonal tensor **Q** *preserves the magnitudes of vectors and the angles between them.*

It can be shown that the proper orthogonal tensor has the representation

$$\mathbf{Q} = \mathbf{E} \cos\omega + (1 - \cos\omega)\mathbf{ee} - \mathbf{e} \times \mathbf{E} \sin\omega.$$

Polar decomposition

A symmetric tensor is said to be *positive definite* if its eigenvalues are all positive.

Any nonsingular tensor **A** *of order two may be written as a product of an orthogonal tensor and a positive definite symmetric tensor. The decomposition may be done in two ways: as a* ***left polar decomposition***

$$\mathbf{A} = \mathbf{S} \cdot \mathbf{Q}$$

or as a ***right polar decomposition***

$$\mathbf{A} = \mathbf{Q} \cdot \mathbf{S}'.$$

Here **Q** *is an orthogonal tensor of order two, and* **S** *and* **S** *are positive definite and symmetric.*

Deviator and ball tensor representation

For **A** we can introduce the representation

$$\mathbf{A} = \frac{1}{3} I_1(\mathbf{A})\mathbf{E} + \operatorname{dev}\mathbf{A}.$$

Such a representation is found useful in the theory of elasticity. Moreover, it is used to formulate constitutive equations in the theories of plasticity, creep, and viscoelasticity.

The tensor dev**A** is defined by the above equality. It has the same eigenvectors as **A**, but eigenvalues that differ from the eigenvalues of **A** by $(1/3)I_1(\mathbf{A})$

$$\tilde{\lambda}_i = \lambda_i - \frac{1}{3}\operatorname{tr}\mathbf{A}.$$