On finding the expression for the elastic moduli linking tension and torsion

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The deformation of thin rods is investigated in this work.

Internal energy: $$\rho_0 u(\underline{e},\underline{k}) = \frac{1}{2}\underline{e}\cdot\underline{A}\cdot\underline{e} + \underline{e}\cdot\underline{B}\cdot\underline{k} + \frac{1}{2}\underline{k}\cdot\underline{C}\cdot\underline{k}$$

There are several elasticity tensors describing deformation of thin rods, which are responsible for:

•the rod tension and laminated shift:

$$\mathbf{A} = A_1\mathbf{d}_1\,\mathbf{d}_1 + A_2\mathbf{d}_2\,\mathbf{d}_2 + A_3\mathbf{d}_3\,\mathbf{d}_3,$$

•the rod bending and torsion:

$$\mathbf{C} = C_1\mathbf{d}_1\,\mathbf{d}_1 + C_2\mathbf{d}_2\,\mathbf{d}_2 + C_3\mathbf{d}_3\,\mathbf{d}_3.$$

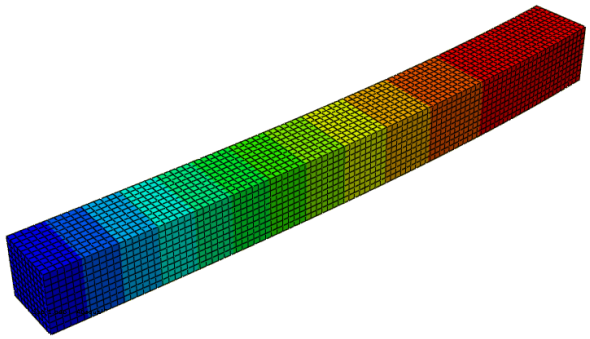
Because of the independence of the above-described tensors on the strain , they can be found according to the linear theory.

The main idea is to have the angular momentum and the linear momentum received according to the rods' theory in line with ones received from the modeling in 3d.

$$\rho_0(u + \Theta_1 \cdot \psi) = \int \rho u_{(3)} dxdy$$

$$\rho_0(u \cdot \Theta_1 + \Theta_2 \cdot \psi) = \int \rho a \times u_{(3)} dxdy$$

Where ψ is the rotation angle, u – displacement, Θ_1 and Θ_2 are the axial and polar inertia tensors respectively



In perspective, the research will be devoted to the problem of finding the expression for the third elasticity tensor, which components represent the modules of elasticity, evaluating the connection between tension and torsion.

Neglect of this tensor in simple cases can be justified, because not too strongly curved rods can be considered as straight rods. On the other hand, in comparison with some of the accurate three-dimensional elasticity solutions, it became obvious that one cannot further ignore this tensor in complex cases , especially when determining the displacements. Main difficulty in determining the elastic modulus connected with the fact that they depend on the cross sectional shape of the rod.

Thank you for your attention!