Internship in Saint Petersburg

September 18th, 2015

Mona Dannert

Supervisor: Vitaly Kuzkin

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I - Introduction

- Master Studies: Computational Engineering at Leibniz University, Hanover

http://www.hannover.de/Wirtschaft-Wissenschaft/Wissenschaft/Hochschulen-und-Forschung/Hochschulen/

I - Introduction

- Master Studies: Computational Engeneering at Leibniz University, Hanover
- Structure of Studies
  - 2 semesters of lectures
  - 1 semester of internship (industry or abroad)
  - 1 semester Master's thesis

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- Abroad: get to know university research

I - Introduction

- Prof. Nackenhorst of Institute of Mechanics and Numerical Mechanics:

  => Department of Theoretical Mechanics

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- Saint Petersburg:
  - arriving: April, 10^{th} 2015
  - departure: October, 10^{th} 2015
  - research on dynamic buckling of columns

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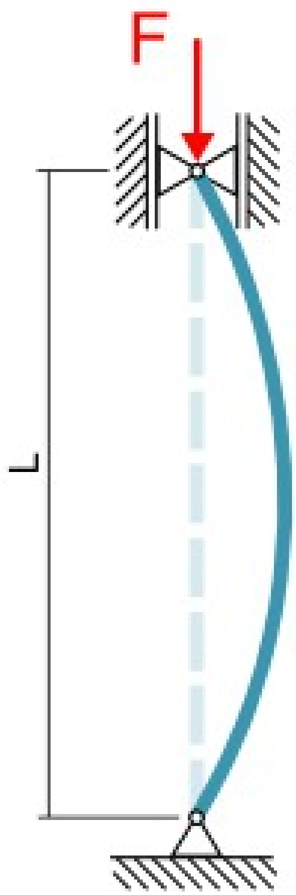
II – Fundamentals of Buckling

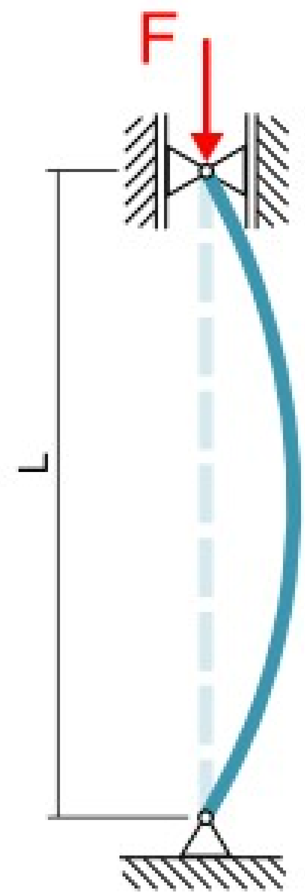
- Statics
  - Euler rod: $$F_E = \frac{\pi^2 EI}{L^2}$$



http://www.maschinenbau-wissen.de/skript3/mechanik/festigkeitslehre/134-knicken-euler

II – Fundamentals of Buckling

- Statics
  - Euler rod: $F_E = \frac{\pi^2 EI}{L^2}$
- Dynamics
  - Buckling load exceeds Euler force



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II – Fundamentals of Buckling

- Dynamics: three possibilities of applying load
  - suddenly applied force
  - moving one end by constant velocity
  - impact

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- Dynamics: three possibilities of applying load
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=> this is what happens in a common testing machine

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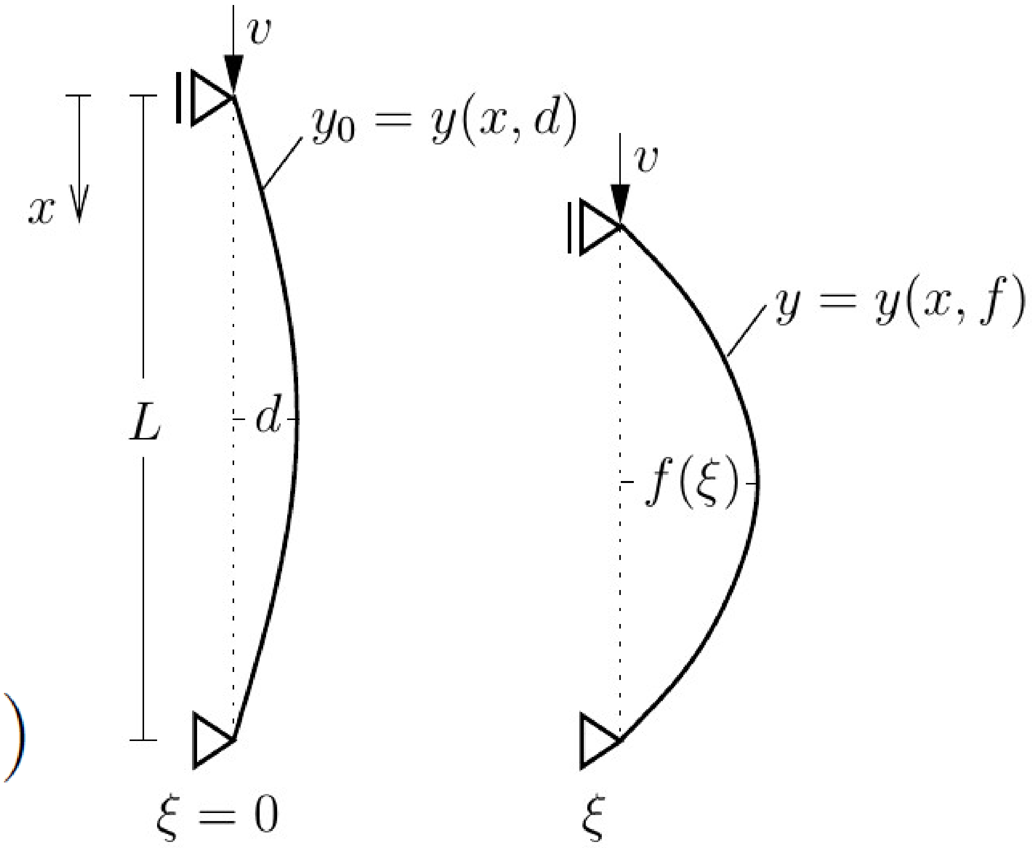
III – Hoff's Problem

- Published in 1951
- Differential equation for dynamic buckling of an initially curved column
  - Describing a rod compressed by constant velocity
  - Influence of just two parameters

III – Hoff's Problem

- Initially curved colum
  - initial deflection d
  - constant velocity v
  - deflection function $y = y(x, f)$
  - dimensionless amplitude $f = f(\xi)$



III – Hoff's Problem

- Transverse motion of the column

$$\frac{\partial^2}{\partial x^2}\left\{EI\left[\frac{\partial^2 y}{\partial x^2}-\frac{\partial^2 y_0}{\partial x^2}\right]\right\}+P\frac{\partial^2 y}{\partial x^2}+\mu A\frac{\partial^2 y}{\partial t^2}=0$$

  - Deflection function $y = y(x,f) = Rf\sin\frac{\pi x}{L}$

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  - Deflection function $y = y(x, f) = Rf \sin\frac{\pi x}{L}$
  - Boundary conditions $y(0,\xi) = y(L,\xi) = 0$

$$\left.\frac{\partial^2 y}{\partial x^2}\right|_{x=0} = \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=L} = 0$$

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  - Deflection function $y = y(x,f) = Rf\sin\frac{\pi x}{L}$

- Compressing Force

$$P=\frac{EA}{L}\left\{vt-\frac{1}{2}\int_0^L\left[\left(\frac{\partial y}{\partial x}\right)^2-\left(\frac{\partial y_0}{\partial x}\right)^2\right]\mathrm{d}x\right\}$$

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- Differential equation of dimensionless amplitude

$$f'' + \Omega \left[\frac{1}{4} f^3 + (1 - \xi) f - \frac{d^2}{4} f - d \right] = 0$$

  - Boundary conditions $f(0) = d \qquad f'(0) = 0$

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  - Similarity number $\Omega = \pi^8 \left(\frac{R}{L} \right)^6 \left(\frac{v_s}{v} \right)^2$

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- Buckling time ξ_* by solving

$$\left. \frac{\mathrm{d}P}{\mathrm{d}\xi} \right|_{\xi=\xi_*} = EA \left(\frac{\pi R}{L} \right)^2 \left[1 - \frac{1}{2} f_* f'_* \right] = 0$$

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$$\frac{P_*}{P_E} = \xi_* - \frac{1}{4}(f_*^2 - d^2)$$

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IV – Analytical Solution

- Assuming small imperfections
- Linearized differential equation

$$f'' + \Omega\left[(1-\xi)f - d\right] = 0$$

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- Solution by using of Airy functions

$$\begin{aligned} f(\xi) = \quad & f_0\,\pi\left[\mathrm{Ai}\left(\Omega^{\frac{1}{3}}(\xi-1)\right)\mathrm{Bi}'\left(-\Omega^{\frac{1}{3}}\right) - \mathrm{Bi}\left(\Omega^{\frac{1}{3}}(\xi-1)\right)\mathrm{Ai}'\left(-\Omega^{\frac{1}{3}}\right)\right] \\ & - d\,\pi\,\Omega^{\frac{2}{3}}\left[\mathrm{Ai}\left(\Omega^{\frac{1}{3}}(\xi-1)\right)\int_0^{\xi}\mathrm{Bi}\left(\Omega^{\frac{1}{3}}(z-1)\right)\mathrm{d}z\right. \\ & \left. - \mathrm{Bi}\left(\Omega^{\frac{1}{3}}(\xi-1)\right)\int_0^{\xi}\mathrm{Ai}\left(\Omega^{\frac{1}{3}}(z-1)\right)\mathrm{d}z\right]. \end{aligned}$$

IV – Analytical Solution

- Using asymptotic formula $\mathrm{Ai}(z) \sim \dfrac{e^{-\frac{2}{3}z^{\frac{3}{2}}}}{2\sqrt{\pi}z^{\frac{1}{4}}}$
  - Assuming $z = \Omega^{\frac{1}{3}}(\xi - 1) > 0 \quad \mathrm{Ai} \rightarrow 0$

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$$f(\xi) \approx \pi\, d\, \Omega^{\frac{1}{3}}\, \mathrm{Bi}\left(\Omega^{\frac{1}{3}}(\xi - 1)\right)$$

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$$f_* = \sqrt{2}\left\{\Omega \, \frac{3}{4} \ln\left[\frac{2}{\pi \, d^2 \, \Omega}\right]\right\}^{-\frac{1}{6}}$$

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=> restriction: $\Omega < \dfrac{4}{\sqrt{3}\pi d^2}$

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- Inserting solution

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into $$\left.\frac{\mathrm{d}P}{\mathrm{d}\xi}\right|_{\xi=\xi_*} = EA\left(\frac{\pi R}{L}\right)^2 \left[1 - \frac{1}{2} f_* f_*'\right] = 0$$

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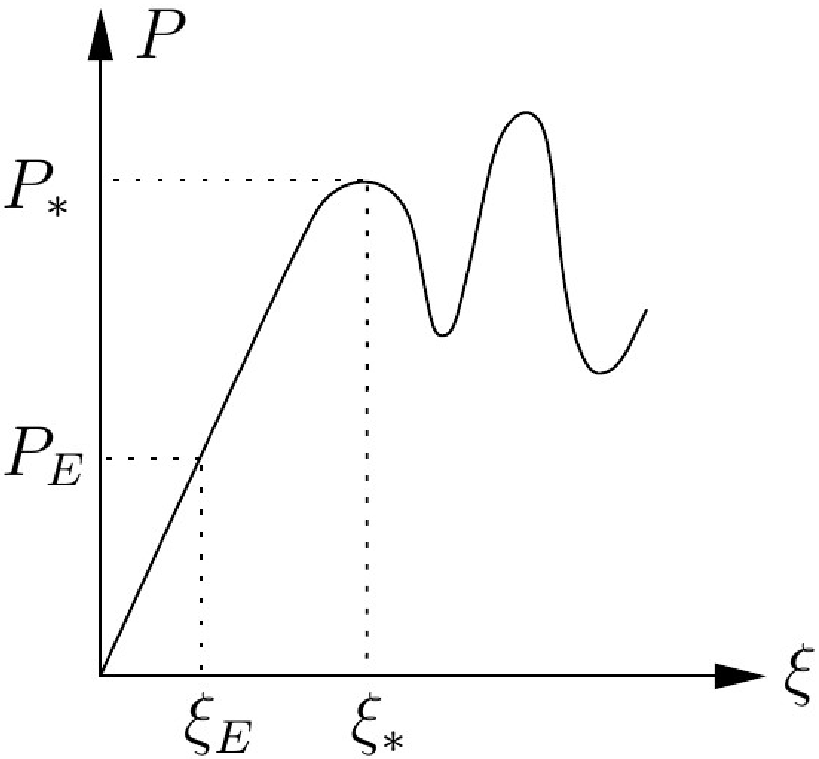
=> Buckling time $$\xi_* = 1 + \frac{1}{\Omega^{\frac{1}{3}}}\left\{\frac{3}{4}\ln\left[\frac{2}{\pi\, d^2\,\Omega}\right]\right\}^{\frac{2}{3}}$$

IV – Analytical Solution

- Ratio of buckling force to Euler force



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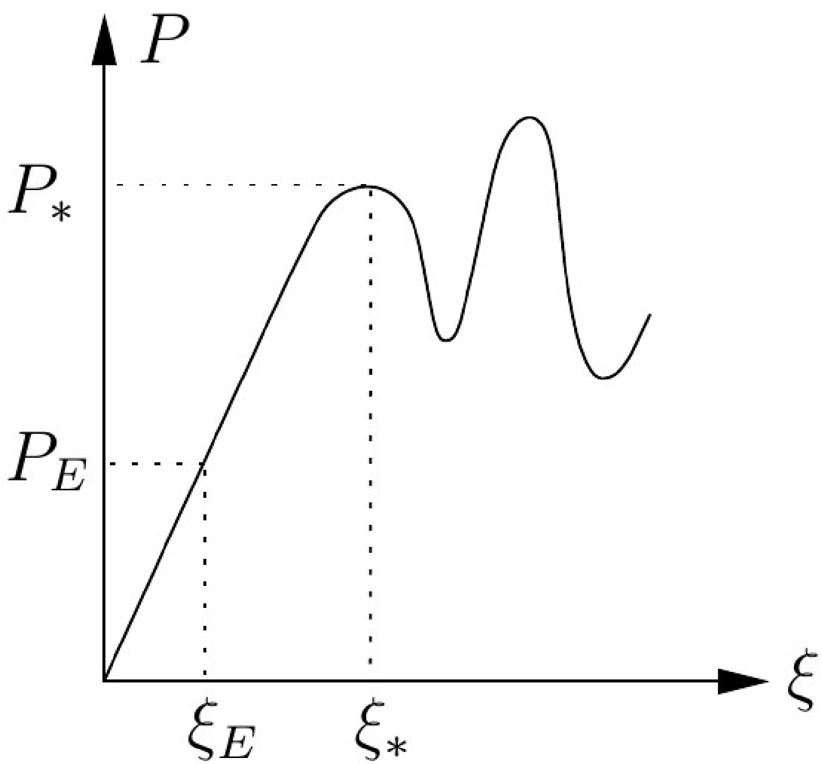

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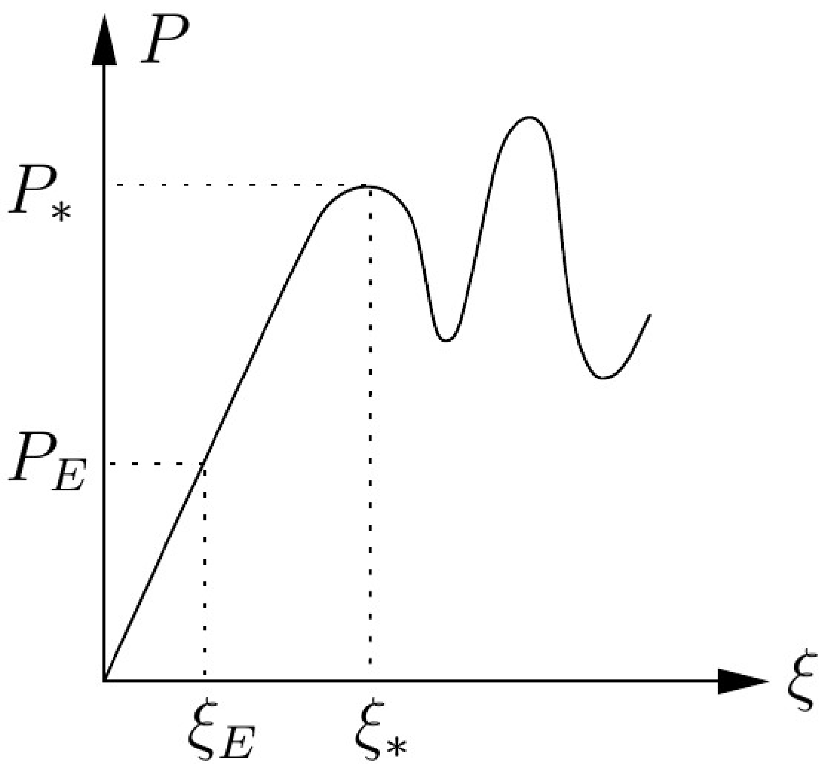

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- small initial defleciton d

- similarity number $\Omega < \dfrac{4}{\sqrt{3}\pi d^2}$

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V – Results

- Computation with matlab

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- Comparison with numerical solution of Hoff's differential equation

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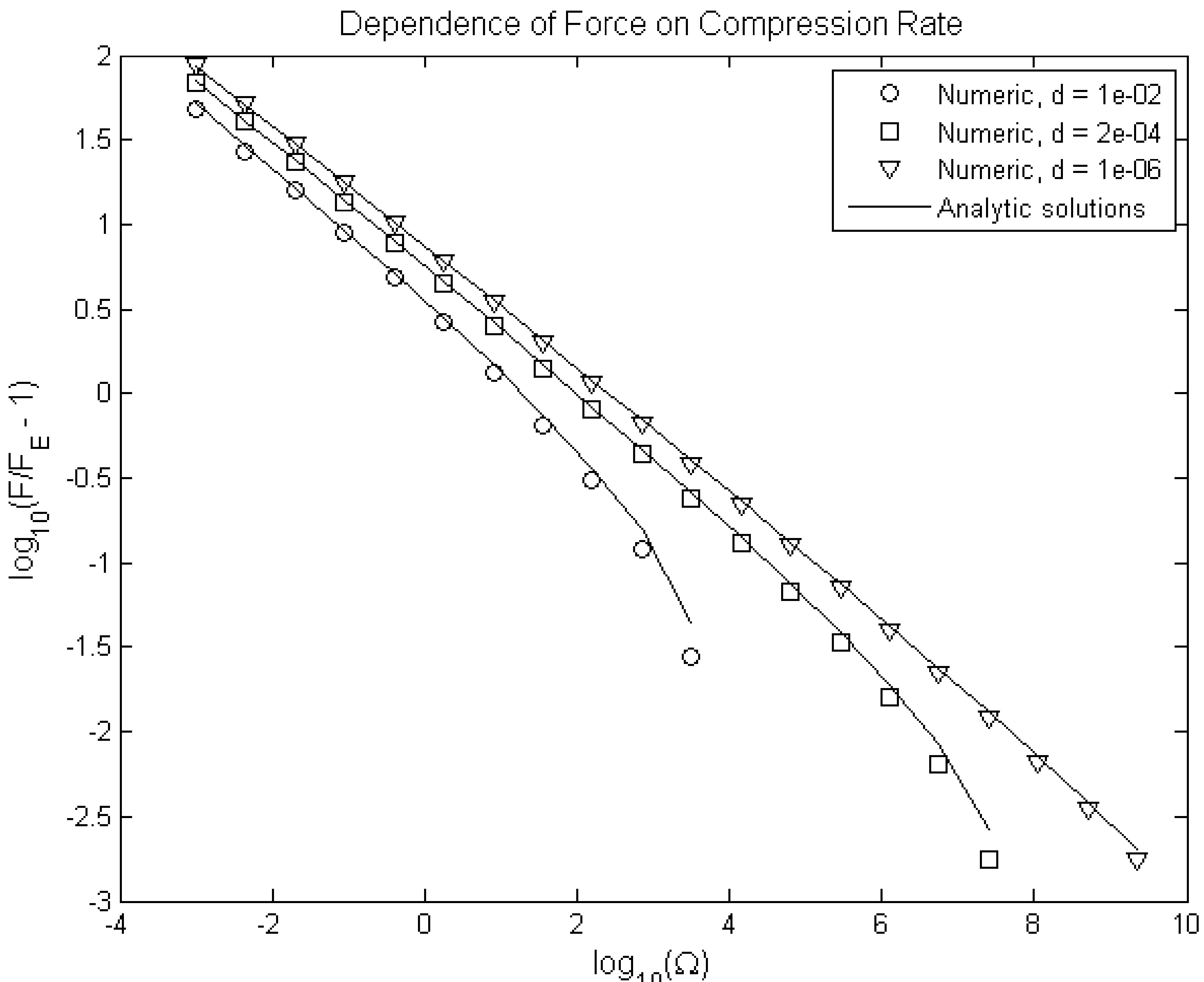

Dependence of Force on Compression Rate
Numeric, d = 1e-02
Numeric, d = 2e-04
Numeric, d = 1e-06
Analytic solutions
log10(F/FE - 1)
log10(Ω)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-4
-2
0
2
4
6
8
10

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VI – Conclusion

- Solving Hoff's problem analytically
  - under certain conditions

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- Solving Hoff's problem analytically
  - under certain conditions
- Analytic solution approximates numeric one the better
  - the smaller initial deflection is
  - the smaller similarity number is
- Dynamic buckling force tends to Euler force for large similarity numbers / small velocities

Literature

Hoff N.J. The dynamics of the buckling of elastic columns. J. Appl. Mech. 1951, Vol. 18, pp. 68–74.

Hoff N.J. Dynamic stability of structures: keynote address at the Int. Conf. on Dynamic Stability of Structures, Northwestern University, October 18, 1965.

Dym C.L., Rasmussen M.L. On a perturbation problem in structural dynamics, Int. J. of Non-Linear Mech., 3 (2), 1968, pp. 215–225.

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