

# Study of the Planet–Satellite System Growth Process as a Result of the Accumulation of Dust Cloud Material

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**Abstract**—In our paper, we suggest a model allowing study of the growth process of the double system (planet–satellite) as a result of the accumulation of scattered material from the common dust condensation. The model consists of two components—a computer component and an analytic component. In the course of the numerical experiment, the computer model allows determination of the dependence of the ratio of particle number captured by each body on their mass ratio. From there, the obtained dependence is used to close the analytical model of the protoplanet growth that is reduced to the solving of a differential equation. It is shown that at any form of the dependence, the equation is integrated in quadratures, and its solution in a number of concrete cases is studied. As a result, possible scenarios of double-system growth are obtained.

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## INTRODUCTION

The problem of the formation of the planetary Earth–Moon system remains one of the serious questions of modern cosmochemistry and astrophysics. In the western literature, the most popular hypothesis is the Giant Impact Hypothesis proposed in the middle of the 1970s by a group of American scientists (Hartmann and Davis, 1975; Cameron and Ward, 1976) and according to which at the final stage of the accumulation the Earth collided with a body of planet size. As a result of the collision, the molten material of the Earth's mantle was ejected to the circumterrestrial orbit, where it quickly accumulated in the form of the Earth's satellite, the Moon. Subsequent calculations showed (Melosh and Sonett, 1986; Canup and Righter, 2000) that in the giant impact, close to 80% of the ejected material originated at the expense of an unknown impactor, and not from the Earth's material. It was shown (Galimov, 2004) that a number of geochemical observations do not agree with the Giant Impact Hypothesis.

In (Galimov, 1995) an alternative scenario was proposed. According to it, the Moon and Earth originated as a double system from the collapse of a cloud of heated dust particles of primary composition. This scenario is based on the idea of the possibility of planet–satellite system formation in the process of the accumulation of gas-dust condensations (Gurevich and Lebedinskii, 1950; Eneev and Kozlov, 1977). In (Galimov and Krivtsov, 2005) a new model of the formation of the Moon that took both geochemical and dynamical aspects of the problem into account was

proposed. According to the model of (Galimov and Krivtsov, 2005), the process of formation of the Earth–Moon system can be divided into three stages.

1. The gravitational instability led to the formation of gas–dust condensations that in time formed a rather large and dense cloud that began to collapse under its own gravitation.

2. The formation as a result of the collapse of two hot bodies—embryos of the present-day Earth and Moon.

3. The further growth of embryos at the expense of the material accumulation from interplanetary space.

In (Galimov and Krivtsov, 2005), the second stage of the system formation—the rotational collapse of the gas-dust cloud—was considered in detail. However, for the closure of the model, the third stage—a slow growth of embryos as a result of the material accumulation from the interplanetary space—is significant. Our given paper deals with the study of this process and the obtaining of the answer to the main questions arising—how will the material be distributed among two embryos, and how will their growth and change of relative sizes occur? Answers to these questions are necessary for the further analysis of the geochemical composition of formed bodies and will help in the analysis of the presently existing dynamical and geochemical characteristics of the Earth–Moon system. The embryo growth process was studied earlier by analytical methods (Safronov, 2002; Harris, 1978). In (Harris, 1978), the results for the case when the mass of one of the planet embryos is much less than the mass of another were obtained analytically, and they

are similar to results of our given work. In our paper, the combined computer analytical model of the double-system growth as a result of the accumulation of the scattered material of the dust condensation is proposed; this model is a continuation of the approach set forth in (Galimov and Krivtsov, 2005; Vasilyev et al., 2004).

### STATEMENT OF THE PROBLEM

Let us consider the model problem on the development of a system of two gravitating bodies (it can be applied, in particular, to the development of the Earth–Moon system). Let us assume the initial existence of a pair of two large cosmic bodies—embryos rotating around a common center of mass. Assume that the embryo growth occurs at the expense of the fall of the material particles from the surrounding space on them, and the fall of particles occurs in the gravitational field of the double system. Further, for definiteness, two large bodies will be called protoplanets, and the material falling on them will be called dust particles. These names are rather conditional; in the course of the growth, bodies can turn into a planet and its satellite, which is not, generally speaking, the planet; the falling material can both be in the form of dust particles, and be gas–dust condensations or planetesimals. But for the simplicity of exposition, the indicated terminology will be kept.

Average densities of the material of protoplanets and dust particles are assumed to be equal. Indeed, differences in density can be connected both with the difference in the composition of the material of protoplanets and dust particles, and with the compression of protoplanets under their own gravitation. However, taking these factors into account would greatly complicate the model, and therefore is taken out beyond the limits of our paper.

Let us denote masses of protoplanets by  $m_1$  and  $m_2$  (for the definiteness, we will assume  $m_1 > m_2$ ),  $m = m_1 + m_2$  is the total mass of the system,  $a$  is the distance between protoplanets, and  $G$  is the gravitation constant. In the paper, only the mutual attraction of protoplanets and their gravitational influence on dust particles are taken into account. The gravitational interaction of dust particles and their effect on protoplanets are not taken into account. Radii of protoplanet orbits,  $a_1$ ,  $a_2$ , and the angular velocity  $\omega$  of their rotation around the center of mass of the system are considered to be invariable and are defined by formulas

$$a_1 = \frac{m_2}{m} a, \quad a_2 = \frac{m_1}{m} a, \quad \omega^2 = \frac{Gm}{a^3}. \quad (1)$$

For simplification, the rotation of the two-body system around the Sun is not taken into account in the

given statement of the problem. Since the region under consideration is located within the Hill sphere for the largest of bodies, such simplification is justified. In our paper, comprehensive numerical–analytical modeling of the process of accumulation of particles by embryos is carried out. The purpose of the work is to find out on the basis of computing experiments and analytical calculations at different variances of the statement of the problem and its parameters how the growth of embryos at the expense of particles falling on them will occur. The problem is considered in the two-dimensional and three-dimensional statement. In all numerical experiments, direct calculations of trajectories of each particle were carried out. These trajectories are rather various and complex: a particle can execute several revolutions around the center of mass of the system until it falls on one of the embryo bodies.

### DESCRIPTION OF THE COMPUTER MODEL

The system of two protoplanets that rotate in the plane  $XY$  in circular orbits around a common center of mass is considered. The smaller bodies (particles) that model a cosmic dust cloud surrounding the planet embryos fall sequentially on this system. In the three-dimensional statement, the initial positions of dust particles are uniformly distributed over the cylindrical surface  $C_{Rh}$  with radius  $R$  and height  $2h$ . The axis of the cylindrical surface  $C_{Rh}$  coincides with the common center of circular orbits of main bodies. Masses of particles are equal, and initial velocities are zero. Coordinates of every subsequent particle are chosen randomly on the surface  $C_{Rh}$ , and then its trajectory in the gravitational field of the double system before the contact with the surface of one of the bodies is calculated. The surfaces of protoplanets are modeled by spheres with radii  $r_1$  and  $r_2$ . The computing is stopped if a particle flies out beyond the limits of the sphere of the radius  $2R$  with the center in the center of mass of the system. Then the next particle is brought on the surface  $C_{Rh}$ , and the process is repeated. As a result of the multiple repetition of the calculation, the registration of the number of particles fallen on the first and second body is carried out. In the given statement, this registration of particles describes the change of masses of main bodies. The influence of particles on the motion of main bodies is not taken into account. In the two-dimensional statement, a single difference is that particles are brought onto a circle that is the intersection of the cylinder  $C_{Rh}$  with the plane  $XY$  (Fig. 1).

### ANALYTICAL DESCRIPTION OF THE GROWTH OF PROTOPLANETS

In the statement of the computer experiment described above, the change of masses of planet

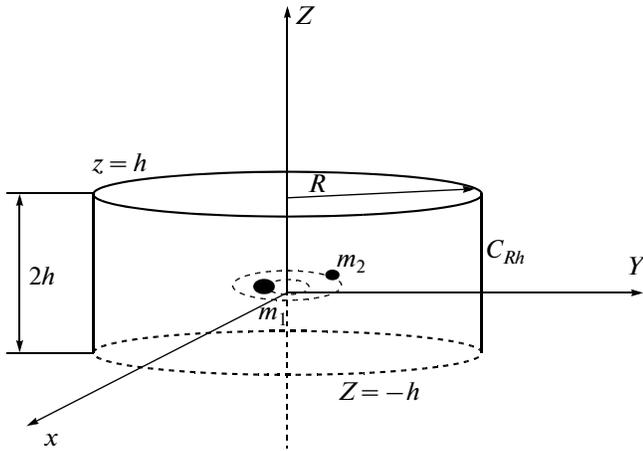


Fig. 1. Geometrical model of the system of two rotating embryos.

embryos is not taken into account, and only the number of particles,  $n_1$  and  $n_2$ , fallen on the first and second body, respectively, is determined. However, these data allow the construction of the analytical model of the growth of protoplanets. Let us show this. From the computer calculation, the function  $f$  that connects the mass ratio  $m_2/m_1$  of protoplanets with the proportion of particles fallen on them,  $n_2/n_1$ , can be obtained:

$$f\left(\frac{m_2}{m_1}\right) = \frac{n_2}{n_1}. \tag{2}$$

Let us introduce a dimensionless parameter equal to the mass ratio of protoplanets

$$\xi = \frac{m_2}{m_1}, \quad 0 < \xi < 1. \tag{3}$$

Obviously, the function  $f(\xi)$  have to have the following properties:

$$0 < f(\xi) < 1; \quad f(0) = 0, \quad f(1) = 1. \tag{4}$$

Now let us assume that the fall of particles lasts over a long period of time, considerably exceeding the computer calculation time. Then masses of protoplanets will begin rising, and the ratio of mass increase rates of

protoplanets  $\frac{\dot{m}_2}{\dot{m}_1}$  will be equal to the ratio  $n_2/n_1$  (on condition that densities of the material of protoplanets and dust particles are equal). As a result, the following system of equations can be written for the growth of planet embryos

$$\frac{\dot{m}_2}{\dot{m}_1} = f\left(\frac{m_2}{m_1}\right), \quad m_1 + m_2 = m(t), \tag{5}$$

where  $m(t)$  is the total mass that we will assume to be a known function of time.

Using the obtained equations, let us study how the mass ratio of protoplanets  $\xi$  will change with time.

With this purpose, let us calculate the derivative  $\dot{\xi} = \left(\frac{\dot{m}_2}{m_1}\right)$ , by using equation (5) and expressing masses of protoplanets in terms of  $m$  and  $\xi$ . As a result, we will obtain

$$\dot{\xi} = \Phi(\xi) \frac{\dot{m}}{m}, \tag{6}$$

where the function  $\Phi(\xi)$  is defined by the relation

$$\Phi(\xi) = (1 + \xi) \frac{f(\xi) - \xi}{f(\xi) + 1} \equiv \left(\frac{1}{1 + \xi} + \frac{1}{f(\xi) - \xi}\right)^{-1}. \tag{7}$$

The differential equation (6) is easily reduced to quadratures

$$\int_{\xi_0}^{\xi} \frac{d\xi}{\Phi(\xi)} = \ln \frac{m(t)}{m_0}, \tag{8}$$

where  $\xi_0$  and  $m_0$  are the initial values of corresponding quantities. After the calculating of integral (8) and the finding of  $\xi(t)$ , masses of protoplanets can be calculated by formulas

$$m_1 = \frac{m}{1 + \xi}, \quad m_2 = \frac{\xi m}{1 + \xi}. \tag{9}$$

However, the important conclusions on the dependence behavior  $\xi(t)$  can be drawn without calculating the integral (8). Thus, it follows from equations (6), (7) that the sign of  $\dot{\xi}$  coincides with the sign of the difference  $f(\xi) - \xi$  (all other multipliers are positive, since according to the statement of the problem, masses of protoplanets increase with time). The dynamic equilibrium  $f(\xi) = \xi$  (masses of protoplanets rise with time, but their ratio remains invariable) corresponds to the case  $\dot{\xi} = 0$ . In order to obtain a condition of stability of some equilibrium position, let us variate the equation (6) in the neighborhood of  $\xi = \xi_*$ , this gives

$$\delta \dot{\xi} = \Phi'(\xi_*) \frac{\dot{m}}{m} \delta \xi, \tag{10}$$

where  $\delta \xi$  is the variation (the deviation from the equilibrium position). It follows from the obtained equation that at nonzero  $\Phi'(\xi_*)$ , the necessary and sufficient condition of stability is the condition

$$\Phi'(\xi_*) < 0 \Leftrightarrow f'(\xi_*) < 1. \tag{11}$$

The second inequality from (11) is obtained using the identity  $f(\xi_*) = \xi_*$ . For the analysis of the change of the quantity  $\xi$  with time, it is convenient to examine dependences  $f = f(\xi)$  and  $f = \xi$  on the one diagram— Fig. 2. If the curve  $f = f(\xi)$  is located above the straight line  $f = \xi$ , then the quantity  $\xi$  increases; otherwise, it decreases. A point on the curve  $f(\xi)$  corresponds to a

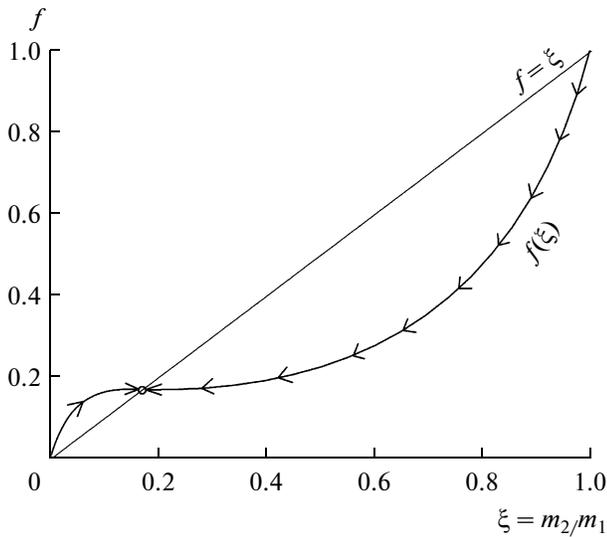


Fig. 2. Diagrammatic representation of the dependence of the ratio of the captured particle number on the mass ratio of protoplanets.

state of the system; with the course of time, the position of this point changes in accordance with the direction of change of  $\xi$ . The point of dynamic equilibrium  $f = f(\xi)$  corresponds to the intersection of the curve  $f = \xi$  with the straight line  $\xi = \xi_*$ . It is evident from Fig. 2 that the equilibrium is stable, when deviating from it in the direction of the increase of  $\xi$ , the curve  $f = f(\xi)$  finds itself below the straight line  $f = \xi$ ;

otherwise, the equilibrium is unstable. Obviously, this condition is equivalent to the stability criterion obtained above (11).

Thus, if the function  $f(\xi)$  is obtained from the computer modeling, then the evolution of the system can be described analytically using equations (6)–(8) or analyzed graphically on the basis of Fig. 2.

RESULTS OF THE COMPUTER MODELING

According to the analytical consideration performed above, the problem of the numerical modeling is reduced to the determination of the function  $f(\xi)$

that connects the mass ratio  $\xi = \frac{m_2}{m_1}$  of protoplanets with the proportion of particles fallen on them,  $n_2 = n_1$ .

The typical form of the dependence  $f(\xi)$  obtained as a result of the computer modeling is presented in Fig. 3. It is evident from the figure that in almost the entire range of values,  $f(\xi) < \xi$  holds. Consequently, the mass ratio decreases with time—the smallest body accumulates considerably less dust particles than the largest one (not in proportion to their sizes), and, as a result, its relative mass decreases. However, this process cannot last to infinity, at some small value  $\xi = \xi_*$ . The curve  $f = f(\xi)$  intersects the straight line  $f = \xi$ , and the dynamic equilibrium comes—masses increase, and the proportion between them does not change. At the same time,  $f'(\xi_*) < 0$  i.e., the situation is analogous to the situation depicted in Fig. 2—the equilib-

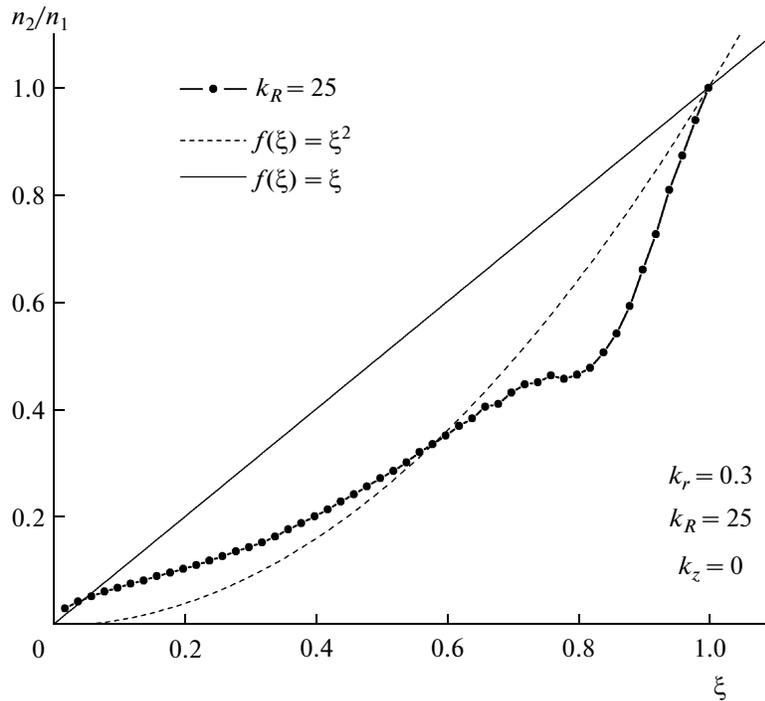


Fig. 3. Typical form of the dependence obtained from the computer experiment.

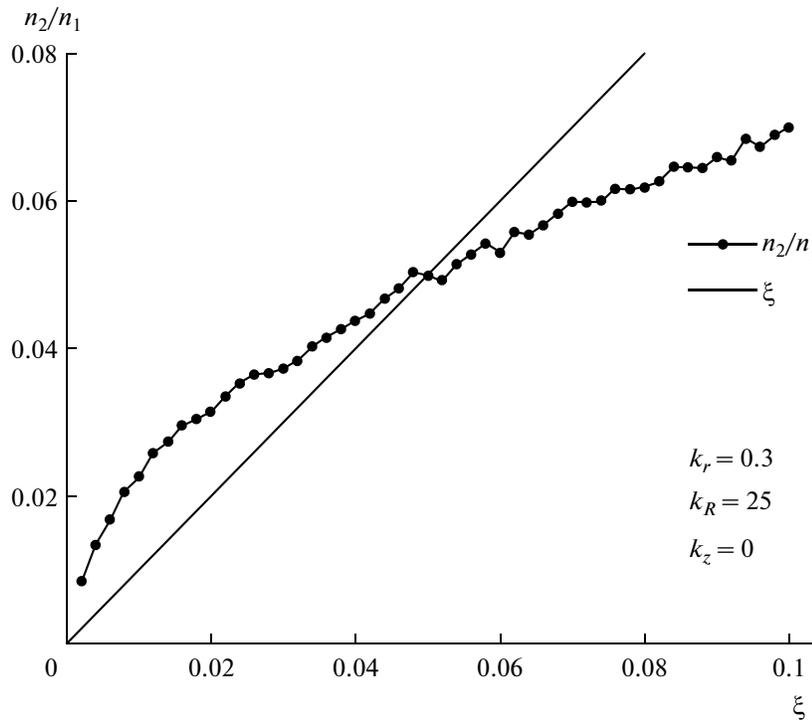


Fig. 4. Case of small ratios of masses of protoplanets.

rium is stable. This intersection is shown in greater detail in Fig. 4; one can see that the equilibrium comes at  $\xi_* = 0.05$ .

Thus, under the unlimited supply of mass to the system of rotating bodies, the ratio of their masses will tend to a small but still fixed value.

However, the system under consideration contains three considerable dimensionless parameters; their values influence on the result

$$k_r = \frac{r_1}{a}, \quad k_R = \frac{R}{a}, \quad k_z = \frac{h}{R}, \quad (12)$$

where  $a$  is the distance between bodies,  $r_1$  is the radius of the biggest body, and  $h$  and  $R$  are the height and radius of the cylinder  $C_{Rh}$ , on which the initial positions of dust particles are specified. The dimensionless parameter  $k_r$  characterizes the remoteness of protoplanets from each other, the parameter  $k_R$  characterizes the remoteness of the initial position of particles, and the parameter  $k_z$  characterizes the relative width of the coordinate  $z$  spread of particles (the thickness of the dust condensation). At  $k_z = 0$ , the three-dimensional statement is equivalent to the two-dimensional one. Results, presented in Fig. 3, correspond to the following values of parameters

$$k_r = 0.3; \quad k_R = 25; \quad k_z = 0. \quad (13)$$

In this and all subsequent computer experiments, the particle number  $n = 10^5$  was used. At such particle number, rather smooth graphs of the dependence  $f(\xi)$

are obtained, and the further increase of the particle number has little influence on the result.

Let us analyze the influence of dimensionless parameters on results of the calculation. The parameter  $k_R$  is the remoteness of the initial position of particles. In Fig. 5, a convergence of results under the increase of  $k_R$  is studied. Values of other parameters are:  $k_r = 0.3$ ,  $k_z = 0$ . Calculations have shown that the convergence of the functional dependence occurs approximately at  $k_R = 25$ ; the further increase of this parameter has little influence on the result. Thus, this value is the maximal one, which it makes sense to use in calculations.

The parameter  $k_r$  is the remoteness of protoplanets from each other. In contrast to  $k_R$ , the parameter  $k_r$  characterizes the physical side of the problem, rather than the computational one. Figure 6 shows results of the variation by this parameter at  $k_R = 25$ ,  $k_z = 0$ . One can see that the smoothest dependence  $f(\xi)$  is realized at  $k_r = 0.45$ , i.e., when protoplanets are initially at a short distance from each other. This is possibly connected with the decrease of the probability of flight of falling particles between protoplanets. As  $k_r$  decreases, the dependence  $f(\xi)$  becomes less regular, and oscillations arise. In Fig. 6, the graph of the ratio of the ejected particle number ( $n_3$ ) to the total number of fallen particles ( $n$ ) is shown as an illustration to the calculation.

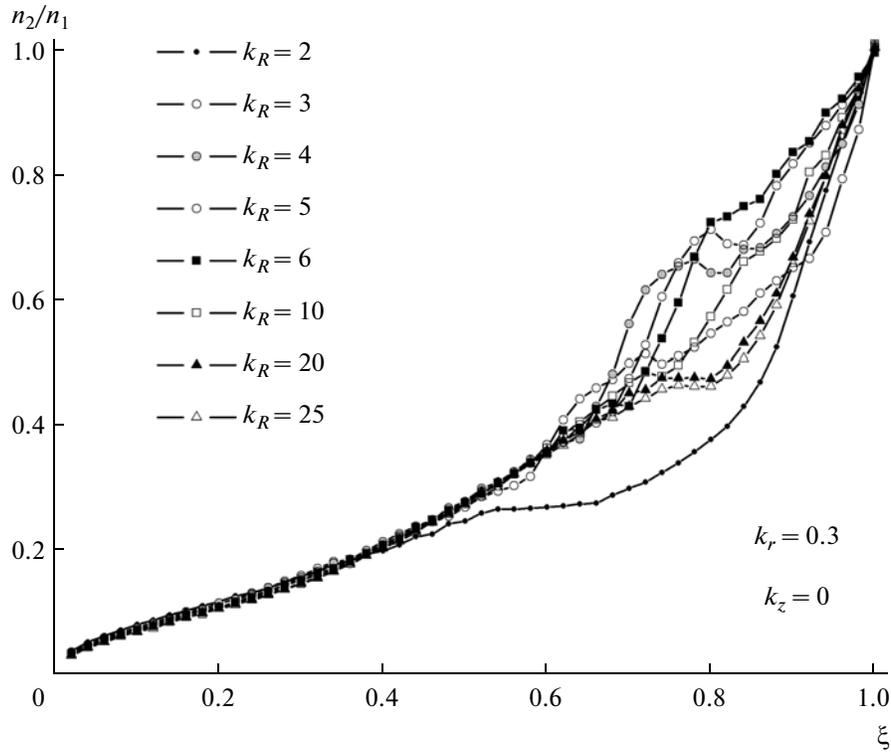


Fig. 5. Variation by the parameter  $k_R$  characterizing the relative size of the feeding region.

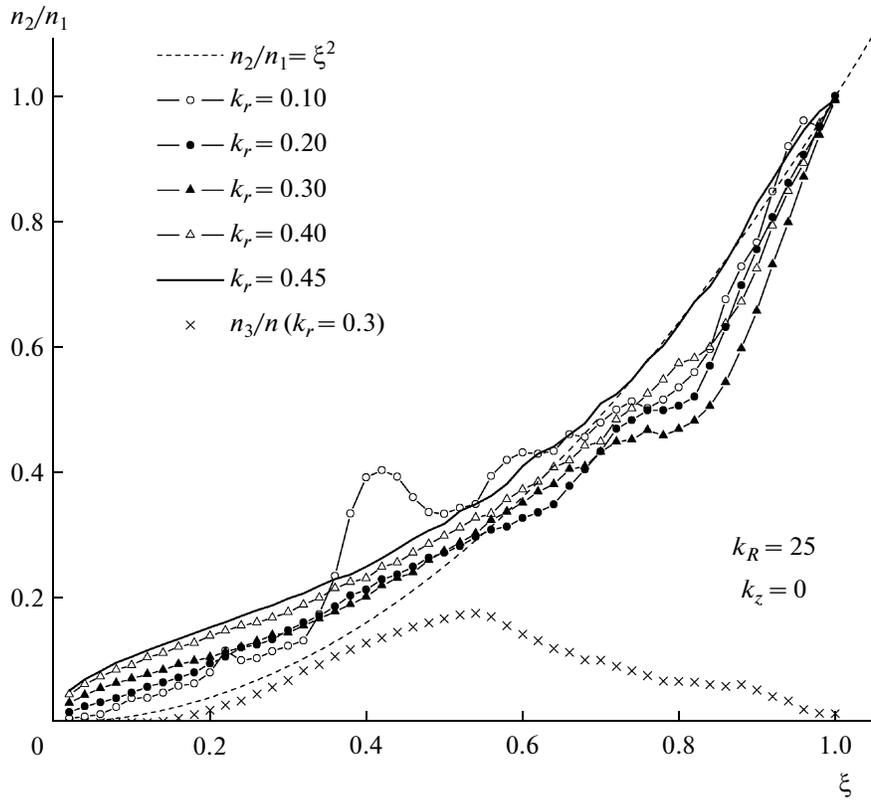


Fig. 6. Variation by the parameter  $k_r$  characterizing the size of protoplanets with respect to the distance between them.

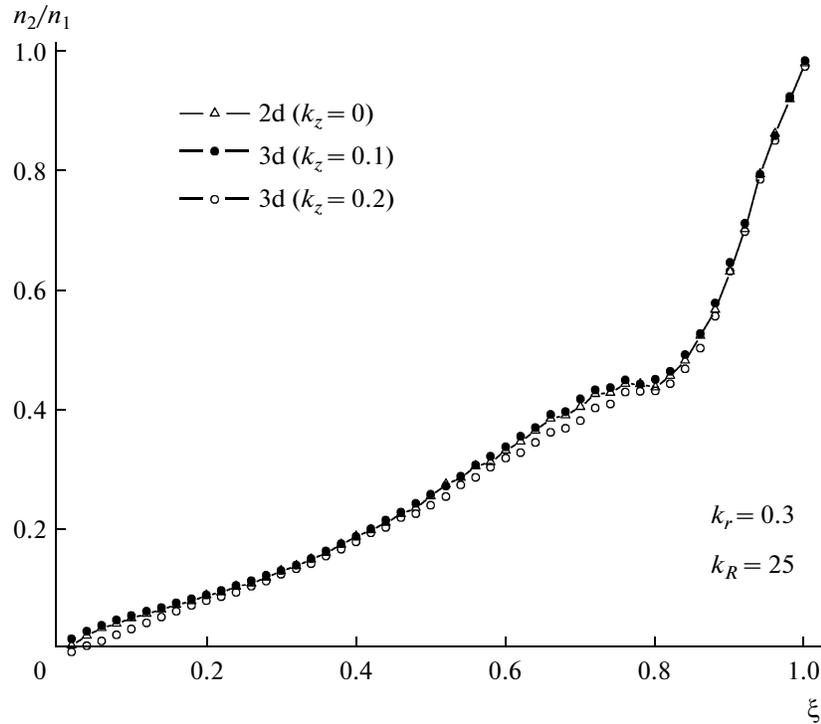


Fig. 7. Comparison of results of the two-dimensional and three-dimensional modeling.

The value  $k_r = 0.3$  was chosen as the base value for the majority of further calculations; this allows us to obtain the rather regular dependence  $f(\xi)$  and is in a good agreement with results of the previous stage of studies on the modeling of double system formation in the process of the rotational collapse of the gas–dust cloud (Galimov and Krivtsov, 2005).

The parameter  $k_z$  is the relative width of the coordinate  $z$  spread of particles (the relative thickness of the particle cloud). Calculations were carried out at  $k_R = 25$ ,  $k_r = 0.3$ . Figure 7 shows the comparison of results obtained in the two-dimensional statement ( $k_z = 0$ ) and in the three-dimensional statement ( $k_z > 0$ ).

One can see that at small  $k_z$ , the functional dependence in the three-dimensional statement differs little from the functional dependence in the two-dimensional statement. But the more detailed consideration reveals that in the region of small values, the influence of the parameter  $k_z$  is considerable. It is evident from Fig. 8 that as the parameter  $k_z$  increases, the point of intersection of the curve  $f = f(\xi)$  with the straight line  $f = \xi$  moves to the left along the axis. In other words, the equilibrium state in the relative increase of masses of protoplanets is reached at smaller ratios  $m_2/m_1$  than in the two-dimensional modeling. When the parameter  $k_z$  increases further, this intersection is no longer observed. In this case, the ratio of the mass of the smallest body to the mass of the biggest one will tend

to zero under the unlimited supply of mass to the system of rotating protoplanets.

Figure 9 shows the change of the dependence  $f(\xi)$  under the considerable increase of the parameter  $k_z$ . It follows from the figure that as the cloud thickness increases, the functional dependence keeps its general form, but becomes more monotonic, and knees disappear; this allows approximating it by a power function.

As a whole, the numerical experiment has shown that under the variation of dimensionless parameters in the wide range, the dependence  $f = f(\xi)$  remains smooth and lies below the straight line  $f = \xi$ , at least for not too small values. Hence, as the accumulation of dust particles goes on, the nonuniform growth of protoplanets occurs—the ratio of their masses  $\xi = m_2/m_1$  decreases. When a sufficiently small value is reached, the dynamic equilibrium—the proportional growth of protoplanets—can come.

### ANALYTICAL APPROXIMATION OF NUMERICAL RESULTS

The consideration of results of the numerical experiment shows that in many cases (see Figs. 7, 10) the function  $f(\xi)$  can be sufficiently well approximated by a power law

$$f = \xi^{k+1}; \quad k > 0. \tag{14}$$

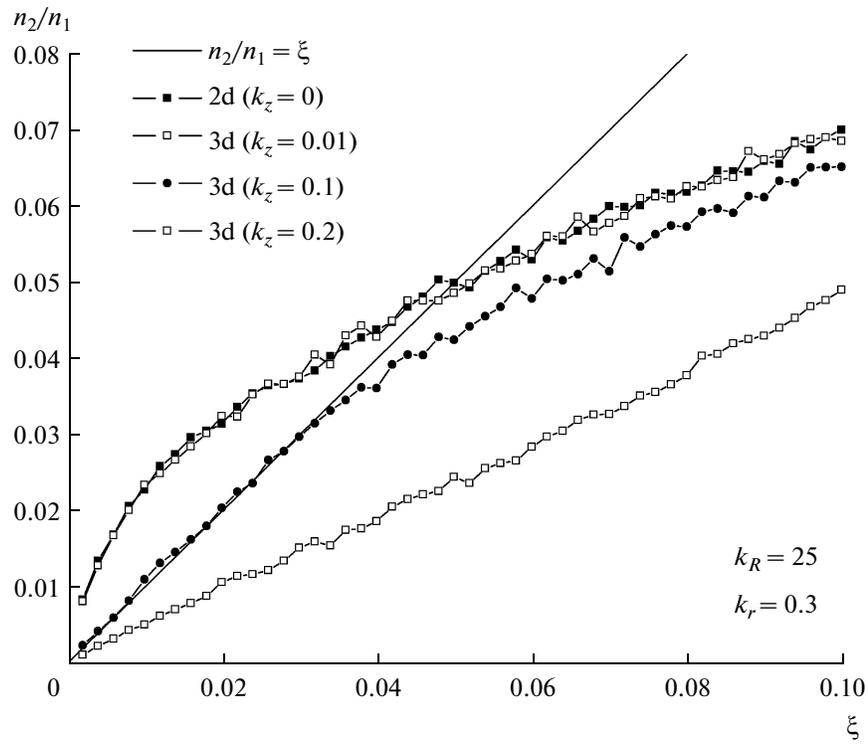


Fig. 8. Comparison of results of the two-dimensional and three-dimensional modeling at small ratios of masses of planet embryos.

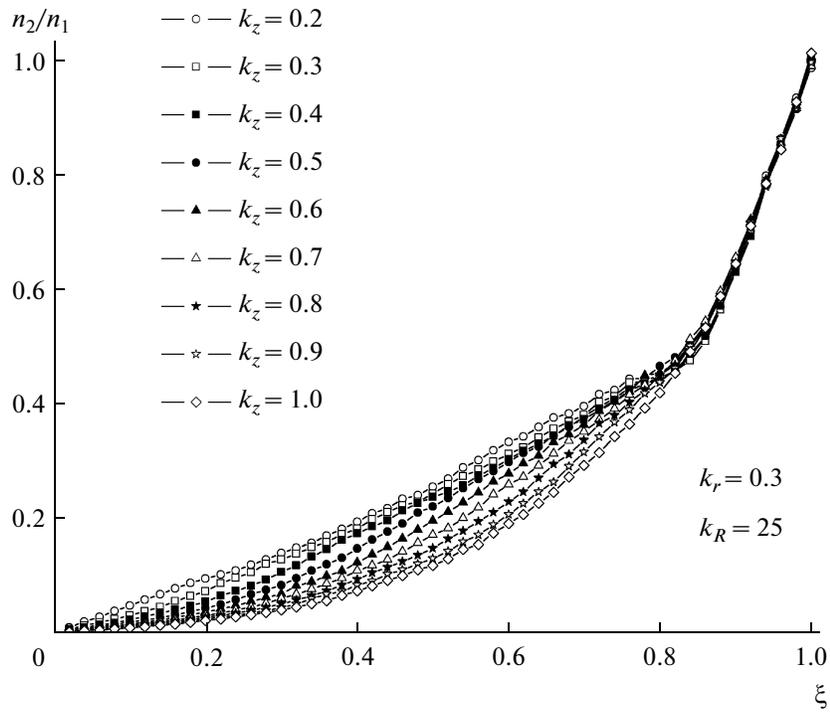


Fig. 9. Variation by the parameter  $k_z$  characterizing the relative thickness of the dust condensation.

Then the first equation of system (5) is easily integrated, and this leads to the following system of algebraic equations:

$$m_2^{-k} - m_1^{-k} = c^{-k}, \quad m_1 = m_2 = m(t), \quad (15)$$

where  $c$  is the integration constant that has the dimension of mass. The obtained system implicitly specifies the desired functions  $m_1(t)$  and  $m_2(t)$ . If the supply of mass is unlimited, then  $m(t) \rightarrow \infty$ , and consequently the mass of the biggest body tends to infinity:  $m_1(t) \rightarrow \infty$ . But then we obtain from the first equation of system (15) that the smallest mass tends toward a constant as follows:

$$m(t) \rightarrow \infty \Rightarrow m_2(t) \rightarrow c = (m_{20}^{-k} - m_{10}^{-k})^{-\frac{1}{k}}, \quad (16)$$

where  $m_{10}$  and  $m_{20}$  are masses of protoplanets at the initial time. Thus, under the unlimited supply of mass, the biggest body accumulates almost all mass of the cloud, whereas sizes of the smallest body remain to be limited. The same conclusions can be obtained by calculating the integral in equation (8); this leads to the following implicit dependence on  $t$ :

$$\frac{1 + \xi}{\xi} \sqrt{1 - \xi^k} = \frac{1}{c} m(t). \quad (17)$$

Under the unlimited supply of mass, the equation (17) gives

$$m(t) \rightarrow \infty \Rightarrow \xi \rightarrow \frac{m(t)}{c}, \quad (18)$$

which agrees with the conclusions obtained above.

The obtained formulas get an especially simple form at  $k = 1$  that corresponds to the quadratic function  $f(\xi) = \xi^2$ . In this case the equation (17) can be solved explicitly:

$$\xi(t) = \frac{1}{2} \left( \sqrt{1 + 4 \frac{c^2}{m^2(t)} - 1} \right) \frac{m(t)}{c}, \quad c = \frac{m_{10} m_{20}}{m_{10} - m_{20}}. \quad (19)$$

After finding  $\xi(t)$ , masses of protoplanets  $m_1(t)$  and  $m_2(t)$  are calculated by formulas (9).

Figures 4 and 7 show how much the quadratic function agrees with numerical results. The power approximation of the function  $f(\xi)$  used in this section holds true, as a rule, for not too small values of  $\xi$ . At small  $\xi$ , the dependence  $f = f(\xi)$  can deviate from the power dependence and intersect the straight line  $f = \xi$  (according to results of computer experiments presented above). As the system approaches this point, the ratio  $m_2 = m_1$  stops changing, and later masses  $m_1$  and  $m_2$  go on growing proportionally to each other.

## COMPARISON

An analogous problem of the protoplanet growth was considered by V.S. Safronov in (Safronov, 2002), where results similar to results of our paper were

obtained as a result of purely analytical study. Formally, another problem was considered in Safronov's paper: planets rotate around the Sun within a feeding cloud, but in our problem they rotate around the common center of mass in a feeding cloud. However, in both problems the rotation of the system occurs within a feeding medium around the center of a feeding cloud, and this allows us to compare them.

In (Safronov, 2002) it was shown that as two bodies grow in the feeding medium, the mass difference will increase with time, i.e., the biggest body becomes still bigger, and the biggest body grows faster both absolutely and relatively; i.e., the ratio  $m_2/m_1$  rises. According to (Safronov, 2002), this ratio can rise only up to the value  $10^{-3}$ . Computer experiments carried out in our paper show that in the two-dimensional statement (Fig. 5) the equilibrium comes at  $m_1/m_2 = 50 \times 10^{-3}$ . But in the three-dimensional statement, this value decreases down to magnitudes on the order of  $10^{-3}$  with an increase of the thickness of the feeding dust cloud. Thus, both the qualitative and the quantitative coincidence of results is seen.

## CONCLUSIONS

In our paper, the problem of growth of the system of two planet embryos that rotate around the common center of mass and accumulate the material from the dust cloud surrounding these embryos is studied. The problem was considered in the two-dimensional and three-dimensional statement, and it is shown that at sufficiently small thickness of the dust cloud, results of the two-dimensional and three-dimensional modeling are almost identical. It is discovered that in the two-dimensional statement the following scenario of the protoplanet growth is realized under unlimited supply of mass to the system: both embryos grow boundlessly, and the ratio of their masses tends toward the value at which the smallest body is about 5% of the mass of the biggest body. In the three-dimensional statement, this scenario is also realized at the small thickness of the dust cloud, but the equilibrium ratio of masses of protoplanets decreases with an increase of the cloud thickness. If the ratio of the cloud thickness to its diameter exceeds the critical value approximately equal to 0.1, then the scenario changes; the biggest body grows boundlessly, the smallest one grows up to some limit, and the mass ratio of protoplanets tends to zero. The analytical approximation for the numerical dependence of the ratio of protoplanet growth rates on the ratio of their masses is proposed, which allowed us to obtain the analytical solution of the problem.

Our paper allows us to draw a conclusion that within limits of the scenario with the unlimited growth of both planet embryos, the present-day mass relation in the Earth–Moon system can be explained by the

fact that the supply of mass has stopped for some reason. For other planets of the Solar System that have considerably smaller ratios of masses of satellites to the mass of the planet, the situation can be explained by the longer supply of mass to the system of rotating embryos of these planetary systems.

## REFERENCES

- Canup, R.M. and Righter, K., *Origin of the Earth and Moon*, Univ. Ariz. Press, 2000.
- Cameron, A.G.W. and Ward, W., The Origin of the Moon, *Proc. 7th Lunar Sci. Conf.*, Houston, 1976, pp. 120–122.
- Eneev, T.M. and Kozlov, N.N., Numerical Simulation of Planets and Protoplanets Nebula Formation, *Preprint of Keldysh Institute of Applied Mathematics*, 1977.
- Galimov, E.M., The Problem of the Moon Origin, in *Osnovnye napravleniya geokhimii* (The Main Directions of Geochemistry), Moscow: Nauka, 1995, pp. 8–45.
- Galimov, E.M., On the Moon Matter Origin, *Geokhim.*, 2004, no. 7, pp. 691–706.
- Galimov, E.M., Krivtsov, A.M., Zabrodin, A.V., et al., Dynamic Model of the Earth–Moon System Formation, *Geokhim.*, 2005, no. 11, pp. 1137–1149.
- Gurevich, L.E. and Lebedinskii, A.I., Planets Formation, *Izv. Akad. Nauk USSR. Ser. Fiz.*, 1950, vol. 14, no. 6, pp. 765–775.
- Harris, A.W., Satellite Formation, II, *Icarus*, 1978, vol. 34, pp. 128–145.
- Hartmann, W.K. and Davis, D.R., Satellitesized Planets and Lunar Origin, *Icarus*, 1975, vol. 24, pp. 504–515.
- Melosh, H.J. and Sonett, C.R., When Worlds Collide: Jetted Vapor Plumes and the Moon's Origin, *Origin of the Moon*, Hartmann, W.K., Phillips, R.J., and Taylor, G.J., Eds., Houston: Lunar Planet. Inst., 1986, pp. 621–642.
- Safronov, V.S., *Izbrannye trudy* (Selected Works), vol. 1: *Proiskhozhdenie Zemli i planet* (Origin of the Earth and Planets), 2002, pp. 163–168.
- Vasilyev, S.V., Krivtsov, A.M., and Galimov, E.M., Modeling Space Bodies Growth by Accumulation of Space Dust Material, *Proc. XXXII Int. Summer School–Conf. Advanced Problems in Mechanics*, St. Petersburg, 2004, pp. 425–429.
- Vityazev, A.V., Pechernikova, G.V., and Safronov, G.V., *Planety zemnoi gruppy: Proiskhozhdenie i rannaya evolyutsiya* (Terrestrial Planets. Origin and the Early Evolution), Moscow: Nauka, 1990.