Review on Bell Inequalities, Entanglement and Decoherence in Particle Physics

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Abstract

Entanglement constitues a key feature of Quantum Mechanics (QM), not only will understanding it give a deeper insight in fundamental features of nature, but it also will yield the way for future applications, for one quantum computation and quantum communication, as well as applications which are not within the reach of our imagination today. As photon or spin $\frac{1}{2}$ states already are subject to thorough investigations concerning entanglement [1], we shall review to what extent Bell Inequlities, Entanglement and Decoherence maintain validity regarding massive meson-antimeson systems, here we will especially focus on the neutral Kaon system. The main motivation for investigating neutral meson-antimeson systems lies in the hope to close both, locality and detection loopholes [2]. Analogies and Differences will be discussed, and the principles of time evolution and $\mathcal{CP}$ violation as well as their connection to Bell Inequalities will be summarized.

1 Introduction

1.1 Beginning of Quantum Physics

Quantum Physics has its roots in 1838: discovery of cathode rays by Michael Faraday, followed by a number of works concerning black body radiation and energy levels of physical states which lead to the quantum hypothesis by Max Planck in 1900 [3]: $E = h\nu$ The exploration of Quantum machenical behaviour eventually lead to the formulation of the EPR-paradox in 1935 by Einstein Podolsky and Rosen [4]. Since than it has been a neverending quest for quantum physicists to refute local reality theory with a loophole free experiment violating Bell inequality.

1.2 Bell inequality and entanglement

The fundaments of quantum mechanics are based on entanglement, an effect which Einstein himself described as "spukhafte Fernwirkung", i.e. spooky action at distance. Entangled states cannot be described by two seperate wavefunctions but by only one. As a consequence, the notion of local realism has to be abandoned, since the wavefunction contains all states in a specific entanglement, and the single states cannot be acted upon without affecting all other states in the very same entangled state. For the sake of completeness it should be mentioned, that it is possible to measure one state in an entangled three (or more) particle state with leaving the other particles entangled, however the "degree" of entanglement is altered.

1.3 Bell inequality (BI) for photons and spin $\frac{1}{2}$ fermions

There are numerous methods to investigate whether a state is seperable or not. All seperable states can be described by local hidden variable theory (LHV) which fulfill the BI. Thus we shall use Bell inequalities as a reliable test for entanglement. For the sake of completeness the derivation of BI for photons and spin $\frac{1}{2}$ fermions has been included in the appendix and has been taken from [7]. Nevertheless, the three most important inequalities in this context are listed below (a, a', b, b' represent individually measured spin components):
- CHSH-inequlity [6] :

$$S = |E(a,b) - E(a,b')| + |E(a',b') - E(a',b)| \leq 2 \tag{1}$$

- Bell inequlity [7] [1]:

$$\begin{aligned} |E(a,b) - E(a,a')| &\leq 1 + E(a',b) \\ |E(a,b) - E(a,a')| &\leq 2 \pm \{E(a',a') - E(a',b)\} \end{aligned} \tag{2}$$

- Wigner inequality [7]:

$$P(a;b) \leq P(a;a') + P(a';b) \tag{3}$$

1.4 Loopholes

Since the beginning of experiments testing entanglement, it has always been a major task to close loopholes which allow to maintain the possibility of explaining the presumably quantum mechanical effects by LHV, exploiting the shortcomings of experimental setups. The most important loopholes are:

detection loophole: In order to guarantee a violation of Bell's inequality for maximally entangled (non–maximally entangled) states, the detectors must have overall efficiencies of at least 0.83 (i.e.for CHSH-inequality) and 0.67 (i.e. for partially entangled states) [8] so that the experiment does not require an additional "‘fair sampling assumption"’, i.e. representative distribution of undetected states with and without entanglement.

locality loophole: Also known as "‘light-cone-loophole"’. It describes the necessity to make sure that the exchange of information within the duration of one measurement and between the two measuring devices is not possible. Attempts to close it include independent random detector orientations, as introduced by Aspect, and large scale spatial seperation of the detectors up to several hundred meters, as carried out by Weihs at al. in addition to random switching based on another quantum system.

[1]Note: It shall not be subject to this review, however one should keep in mind that BI also produces "‘falls negatives"’, i.e. entangled states, which are describable by LHV theory but not seperable are identified as not entangled. Finding the best possible tests belongs to the research field of entanglement witnesses. [See appendix for a short summary].

1.5 Results

Recent experiments conducted with entangled beryllium ions [9] efficiencies reached 0.97, thereby the detection loophole was successfully closed. Nevertheless, it was not possible to close the locality loophole due to the small spatial seperation between the entangled ions. An experiment with distant entangled photons [11] closing the locality loophole notwithstanding, detection efficiencies were too low to close the detection loophole.[2] To name a few results: The experiment carried out by Aspect at al. [10] gave $S = 2.697 \pm 0.015$, The experiment conducted by Weihs at al. ´mentioned above with the analyzers being 400m apart from each [11] other gave $S = 2.73 \pm 0.02$, violating the CHSH-inequality (eq. 1) by over 35 standard deviations.

2 Neutral K-Meson System

2.1 Finding the Kaon states

The first record of Kaon detection reaches back to 1947, when G. D. Rochester and C. C.Butler claimed to have observed the decay of a neutral particle into 2 pions, and a decay of a charged particle into a pion and another neutral particle. They repeated the experiment in cloud chambers and found a surprising paradox [21]. The rate at which these particles showed traces in the chamber guided them to the conclusion, that these particles must have a strong coupling interaction with matter, comparable to that of pions, although their lifetime proved to be much longer than the expected $10^{-22}s$. Ín 1953 Gell-Mann, Nishijima and Nakano proposed a solution to this dilemma by introducing a strangeness scheme[2]. The puzzling story of Kaons began when two decay processes were observed [17]:

$$\tau^+ \rightarrow \pi^+ + \pi^0 \qquad , \quad \eta = (-)^2, \quad 21\% \tag{4}$$

$$\theta^+ \rightarrow \pi^+ + \pi^0 + \pi^0 \qquad , \quad \eta = (-)^3, \quad 6\% \tag{5}$$

where $P|q\rangle = \eta|q\rangle$ and the percentages give the branching ratios in respect to all decays. It turned out that τ^+ and θ^+ showed the strange behaviour of sharing most of their properties like mass ($\approx 500MeV$) and lifetime ($\approx 10^-8s$). However, as the products of the two decays have different parities, parity conservation suggests that τ^+ and θ^+ have to be different particles. These "‘strange processes"’ were summarized under the τ^+-θ^+-puzzle which was solved in 1956 by Lee and Yang [18] who suggested that as in comparison to strong interaction which conserves parity, weak interaction violates it and as a result τ^+ and θ^+ in (4) and (5) might be the same particle decaying via weak interaction. Today this particle is called the K^+ meson produced due to cosmic ray induced events $\pi^+ + n \rightarrow K^+ + \Lambda$. The four K-mesons K^+,K^-,K^0 and $\overline{K^0}$ are pseudoscalar mesons ($J^P = 0^-$), which means that their quark and antiquark spins are antialigned and thus their total angular momentum is zero.
For experimental purposes kaon and antikaon pairs are produced e.g. by strangeness-conserving decay process: $\phi \rightarrow K^0\overline{K^0}$ where $\phi(1020)$ is commonly either pro-

[2]see appendix: Strangeness number

duced by e^+e^- or $p\overline{p}$ collisions[3], first used in the 1960s (today e.g. at LEAR):

$$p\overline{p} \rightarrow K^- \pi^+ K^0, \quad p\overline{p} \rightarrow K^+ \pi^- \overline{K^0} \tag{6}$$

The ϕ state is a non-strange member of the vector meson nonet ($J^P = 1^-$) with the quark content $s\overline{s}$. The coming sections will focus on the "‘strange"’ behaviour of the K-meson and its consequences.

2.2 Strangeness

Kaons have characteristic strangeness quantum number according to:

$$\begin{aligned} S|K^0\rangle &= +|K^0\rangle = +|\overline{s}d\rangle \\ S|\overline{K^0}\rangle &= -|\overline{K^0}\rangle = -|s\overline{d}\rangle \end{aligned} \tag{7}$$

The strangeness eigenvalues of kaon states are obviously representing the number of "strange" quarks, which among others constitute the kaon states. K-mesons are pseudoscalars, thus the states $|\overline{K^0}\rangle$ and $|K^0\rangle$ change their sign under Parity transformation $\mathcal{P}$, as all pseudoscalars do: $\mathcal{P}|K^0\rangle = -|K^0\rangle$ and $\mathcal{P}|\overline{K^0}\rangle = -|\overline{K^0}\rangle$ Charge conjugation $\mathcal{C}$ exchanges particles with their antiparticles, thus applying a $\mathcal{CP}$ transformation on the neutral kaon states yields:

$$\mathcal{CP}|K^0\rangle = -|\overline{K^0}\rangle, \quad \mathcal{CP}|\overline{K^0}\rangle = -|K^0\rangle \tag{8}$$

These equations allow to define the two eigenstates of $\mathcal{CP}$ transformations:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K^0}\rangle), \quad |K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K^0}\rangle) \tag{9}$$

Applying $\mathcal{CP}$-transformation yields: $\mathcal{CP}|K_1^0\rangle = +|K_1^0\rangle, \quad \mathcal{CP}|K_2^0\rangle = -|K_2^0\rangle$
So for the next section, let us keep in mind that there are two $\mathcal{CP}$ eigenvalues $\lambda_{1,2} = +1, -1$ for the two $\mathcal{CP}$ eigenstates $|K_{1,2}^0\rangle$ which are mixed states of the neutral Kaon system.

2.3 $\mathcal{CP}$ violation

As a first step, let us assume that $\mathcal{CP}$-symmetry is conserved in the decay processes. Two main decay branches can be observed and as the important difference lies in their decay time, the intial states shall be called K_S (short time decay) and K_L (long time decay) [12]:
First decay branch: $\Gamma_1^{-1} \sim \tau_1 = 0.89 * 10^{-10}s$

$$K_S \rightarrow \pi^+ + \pi^-, \quad K_S \rightarrow \pi^0 + \pi^0 \tag{10}$$

and the second decay branch: $\Gamma_2^{-1} \sim \tau_2 = 5.17 * 10^{-8}s$

$$K_L \rightarrow \pi^+ + \pi^- + \pi^0, \quad K_L \rightarrow \pi^0 + \pi^0 + \pi^0 \tag{11}$$

The essential difference between these decay branches also lies in their $\mathcal{CP}$ eigenvalues. While the final two pions state has $\mathcal{CP}$=+1, the final three pions state

[3]Note: m_ϕ is just slightly above $2m_K \sim 996MeV$, however the kaon-antikaon decay branch is predominant due to the suppression of the decay into three pions (ratio: 84% and 15% respectively), a property described by the Okubo-Zweig-Iizuka (OZI) rule [13]

has $\mathcal{CP}$=-1. As a first consequence, in order to conserve $\mathcal{CP}$ symmetry according to the equations above, only K_1^0 can decay into two pions, analog only K_2^0 can decay into three pions. Also, as a second consequence because the decay into three pions obviously requires more energy due to the creation of one additional pion, the K_2^0 states decay about 600 times slower. However, in 1964 James Cronin and Val Fitch of BNL found decays of K_L states into two pions ($\mathcal{CP}$ = +1). Cronin and Fitch received the Nobel Prize in Physics for this discovery in 1980. As measurements became more precise, it was proven that $K_L \to 2\pi$ occur with a small fraction ($|\epsilon| \approx 10^{-3}$) regarding all occurring decay processes. This motivates to set up a new basis :

$$|K_S\rangle = \frac{1}{N}(p|K^0\rangle - q|\overline{K^0}\rangle), \quad |K_L\rangle = \frac{1}{N}(p|K^0\rangle + q|\overline{K^0}\rangle) \tag{12}$$

with $p = 1 + \epsilon$, $q = 1 - \epsilon$ and a normalizing factor $N^2 = p^2 + q^2$. In case of no $\mathcal{CP}$ violation the $|K_{1,2}^0\rangle$ basis is equal to the $|K_{S,L}\rangle$ basis of the long and short lived states.

$|K_{S,L}\rangle$ and $|K_{1,2}^0\rangle$ respectively, represent kaonic qubit states comparable to qubit states in quantum information. As these equations show, there is a slight difference regarding the decay process concerning the amount of Kaons and Antikaons. It is common sense that this slight difference accounts for the matter left over after the matter - antimatter annihilation in the early universe.

2.4 Neutral Kaon mixing

The full importance of kaon mixing will become obvious in connection with the next section, i.e. $\mathcal{CP}$ violation. The underlying theory is based upon the problem how to explain the common presence of two neutral Kaons with opposite strangeness quantum numbers. And suggests that before decaying, when both mesons constitute a common state, both states blend. The theory of neutral particle oscillation was first investigated by Murray Gell-Mann and Abraham Pais. The well known time dependent Schrödinger equation:

$$i\hbar\frac{d}{dt}\psi(t) = H\psi(t) \tag{13}$$

yields, by representing ψ in a basis constituted by ψ_1 and ψ_2 :

$$\psi(t) = U(t)\psi(0) = e^{-iHt}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{14}$$

where H denotes the non-Hermitian "‘effective mass"’ Hamiltonian [12]:

$$H = M - \frac{i}{2}\Gamma \tag{15}$$

The diagonal elements are terms representing the strong interactions which conserve strangeness. It is due to the off diagonal elements based on weak interaction which do not conserve strangeness quantum numbers that the states oscillate into another. If the off diagonal elements are real, the ratio is conserved and the states oscillate back and forth, on the other hand, if the off diagonal elements are imaginary, they describe the gradual conversion of the both states' ratio in favor of one of them or in favor of a fixed mixture of both states.

2.5 Strangeness oscillation

Let now be the non-Hermitian "‘effective mass"’ Hamiltonian H as described above be subject to our investigation. In order to compute the time evolution, H has to be diagonalised, which yields H_{diag} :

$$H_{diag} = \begin{pmatrix} M_S - \frac{i}{2}\Gamma_S & 0 \\ 0 & M_L - \frac{i}{2}\Gamma_L \end{pmatrix} \tag{16}$$

Our new found basis constituted by the mixed states $|K_S\rangle$ and $|K_L\rangle$ satisfy: $|K_{S,L}\rangle = \lambda_{S,L}|K_{S,L}\rangle$ with $\lambda_{S,L} = M_{S,L} - \frac{i}{2}\Gamma_{S,L}$
Although $|K_{S,L}\rangle$ do not constitute an orthonormal basis, that is: $\langle K_S|K_L\rangle \neq 1$, they still constitute a basis of the two state space of $|\overline{K^0}\rangle$and$|K^0\rangle$. Applying the Wigner Weisskopf approximation [12], that is to neglect the interaction of the decay products and to assume a pure beam at $t = 0$, the time dependent Schrödinger equation applied on H_{diag} yields the time evolution:

$$|K_{S,L}\rangle(t) = e^{-iM_{S,L}t}e^{-\frac{1}{2}\Gamma_{S,L}t}|K_{S,L}\rangle \tag{17}$$

which finally gives the time evolution of the $|\overline{K^0}\rangle$and$|K^0\rangle$ system [12], thus:

$$|K^0\rangle(t) = g_+(t)|K^0\rangle + \frac{q}{p}g_-(t)|\overline{K^0}\rangle, \quad |\overline{K^0}\rangle(t) = \frac{p}{q}g_-(t)|K^0\rangle + g_+(t)|\overline{K^0}\rangle \tag{18}$$

with:

$$g_\pm(t) = \frac{1}{2}\left[\pm e^{-i\lambda_S t} + e^{-i\lambda_L t}\right] \tag{19}$$

Assuming a pure $|K^0\rangle$ beam after production at $t = 0$, the probability of finding a $|K^0\rangle$ or $|\overline{K^0}\rangle$ state in the beam is given by [4]

$$|\langle K^0|K^0(t)\rangle|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\Gamma t}cos(\Delta m t)\right] \tag{20}$$

and analog:

$$|\langle\overline{K^0}|K^0(t)\rangle|^2 = \frac{1}{4}\frac{|q|^2}{|p|^2}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\Gamma t}cos(\Delta m t)\right] \tag{21}$$

with $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)$ and $\Delta m = m_L - m_S$.

2.6 Regeneration

A beam constituted by a neutral Kaon system decays in time. According to the decay processes the K_S content disappears after a few τ_S leaving only K_L behind. This can be seen as (if $\mathcal{CP}$ violation is neglected) no more 2π decays occur. By guiding the beam through matter the K_S states can be regenrated. The matter contribution M to the Hamiltonian H in equation (15) with the form

$$M = \begin{pmatrix} \mathrm{M}_f & 0 \\ 0 & \overline{M_f} \end{pmatrix} \tag{22}$$

[4] see appendix: Time evolution of kaon states

,where subscript f denotes *final*, has non-degenerate eigenvalues if the propagation takes place in dense matter. This is due to the quark content of both neutral Kaons. While the $\overline{d}$ quark of the $\overline{K^0} = \overline{d}s$ can interact and annihilate with a d quark of a neutron or proton, which leads to a creation of a hyperon, i.e. baryons with at least one strange quark and three light quarks in total, this is not possible for the K^0 state, since its quark content is $d\overline{s}$, this state only experiences quasi-elastic scattering in matter. A thorough investigation on the connection between matter density and regeneration effect is given by [14].

3 Bell inequality for K-mesons

K-mesons show very promising properties concerning the detection and locality loophole. On one hand, Kaons predominantly experience strong interaction, leaving behind interaction or decay products which can be detected with a very high efficiency, which facilitates to close the detection loophole. On the other hand, right after the production the Kaons fly apart with relativistic velocities which is an additional advantage when dealing with the locality loophole. The particles $|K^0\rangle$ and $|\overline{K^0}\rangle$ of the state [19]:

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\left\{|K^0\rangle_l \otimes |\overline{K^0}\rangle_r - |\overline{K^0}\rangle_l \otimes |K^0\rangle_r\right\}$$
$$= \frac{N}{\sqrt{2}}\left\{|K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r\right\} \tag{23}$$

with $N = \frac{N^2}{2pq}$ (see 44), propagate towards different directions, as already symbolized by the subscripts "'l"' and "'r"', denoting left and right side, respectively. This looks completely familiar as it looks like the fourth Bell state $|\psi^-\rangle$:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left\{|V\rangle_l \otimes |H\rangle_r - |H\rangle_l \otimes |V\rangle_r\right\}$$
$$= \frac{1}{\sqrt{2}}\left\{|L\rangle_l \otimes |R\rangle_r - |R\rangle_l \otimes |L\rangle_r\right\} \tag{24}$$

One major difference though is that as photon states cannot decay, the kaon entangled states have to include a time dependent term. After the production and normalizing the surviving kaon pair this leads to the $\Delta\tau = \tau_l - \tau_r$ dependent state [20]:

$$|\psi(\Delta\tau)\rangle = \frac{1}{\sqrt{1+e^{\Delta\Gamma\Delta\tau}}}\left\{|K_S\rangle_l \otimes |K_L\rangle_r - e^{i\Delta m\Delta\tau}e^{\frac{1}{2}\Delta\Gamma\Delta\tau}|K_L\rangle_l \otimes |K_S\rangle_r\right\} \tag{25}$$

with $\Delta m = m_L - m_S$ and $\Delta\Gamma = \Gamma_L - \Gamma_S$ which again corresponds [5] to the produced entangled 2-photon state (signal and idler, subscript s und i, respectively):

$$= \frac{1}{\sqrt{2}}\left\{|V\rangle_i \otimes |H\rangle_s - e^{i\Delta\phi}|H\rangle_i \otimes |V\rangle_s\right\} \tag{26}$$

where $\Delta\phi$ denotes an adjustable phase.

[5]Note: Both photons in (26) have the same weight, but that's not the case in the 2-kaon state (25).

3.1 Measurement methods

Based upon the presented fundamental relations up until now, two different basis can be considered as potential candidates for measurement. K^0-$\overline{K^0}$ basis (Strangeness measurment) and K_S-K_L basis (lifetime measurement). Due to $\mathcal{CP}$ violation the K_S-K_L basis does not constitue an orthogonal set of states as mentioned in section 2.5, however, as the violating parameter is considerably small, it will be approximated by zero for the next sections. In comparison to photon experiments, kaons show the property to decay. This allows to conduct nature-driven "‘passive"’ measurements on kaon states in comparison to "‘active"’ measurements, where the experimenter decides, which basis he wants to measure.

3.1.1 Active measurement

The experimenter decides what he wants to measure.

Strangeness: As discussed in section 2.6 a slab of matter inserted into the beam can "‘filter"’ the $|\overline{K^0}\rangle$, thereby playing the role of a polarizer in analogy to the same situation in photon experiments. In order to achieve high detection efficiencies, either an infinitely dense absorber material is needed, or ultrarelativistic kaons, which by Lorentz-contraction see the absorber material as extremely dense, thus making kaon-nucleon strong interaction much more likely than weak decays [2]

Lifetime: On the other hand, the decision to let the Kaons propagate freely through space and measuring their decay time is equivalent to a measurement in K_S-K_L basis. With the time evolution given by (17), one only has to set a time limit $\tau + \Delta\tau$, until which processes are identified as $|K_S\rangle$ decays, or $|K_L\rangle$ else. [2] states the misidentification probability is reduced to 0.8% when $\Delta\tau = 4.8\tau_S$ and by using detectors with best possible resolution, almost ideal efficiencies are achievable.

3.1.2 Passive measurement

The experimenter remains passive and only examines the decay products.

Strangeness: One can identifiy $|K^0\rangle$ and $|\overline{K^0}\rangle$ with the semileptonic decay[6]:

$$|K^0\rangle \rightarrow \pi^- + l^+ + \nu_l, \quad and \quad \overline{K^0} \rightarrow \pi^+ + l^- + \overline{\nu_l} \tag{27}$$

with $l = e, \mu$. However, detection efficiency for passive strangeness measurement is rather low, due to $|K_S\rangle$ and $|K_L\rangle$ states' semileptonic decay ratios, which are $\simeq 0.66$ and $\simeq 1.1 * 10^{-3}$, respectively [2].

Lifetime: As shown by equation (10) and (11), by additionally neglecting $\mathcal{CP}$ violation the $|K_S\rangle$ and $|K_L\rangle$ states are can be identified by there nonleptonic -by contrast to the semileptonic- decay products, i.e. 2π or 3π.

[6]Semileptonic decay: decay of a hadron through weak interaction into a lepton, its neutrino and another hadron with the rule $\Delta S = \Delta Q$

3.2 Testing BI

3.2.1 Summary

Three different bases for neutral 2-kaon systems have been introduced [7], all of which have corresponding 2-photon states.

KAON STATE	QUASISPIN STATE	PHOTON STATE
$\|K^0\rangle$ $\|\overline{K^0}\rangle$	$\|\uparrow\rangle_z$ $\|\downarrow\rangle_z$	$\|V\rangle$ $\|H\rangle$
$\|K_S\rangle$ $\|K_L\rangle$	$\|\rightarrow\rangle_y$ $\|\leftarrow\rangle_y$	$\|L\rangle$ $\|R\rangle$
$\|K_1^0\rangle$ $\|K_2^0\rangle$	$\|\nwarrow\rangle_x$ $\|\nearrow\rangle_x$	$\|-45°\rangle$ $\|+45°\rangle$

The three inequalities: CHSH-inequalitiy eq.(1), BI eq.(2) and Wigner like BI eq.(3), as well as the various measurement methods (active, passive, lifetime, quasispin) and the known $|K^0\rangle$ and $|\overline{K^0}\rangle$ production processes, their time evolution and representations by $|K_S\rangle$ and $|K_L\rangle$ can now be combined to test for entanglement.

3.2.2 Implementation

Two methods shall be summerized: Time measurement with fixed quasispin and quasispin measurement at a fixed time.

- Time measurement

  A general form of CHSH-inequalitiy eq.(1) which gives freedom of choice in time ($t_i, i = 1,2,3,4$) and quasispin ($k_q, q = n, n', m, m'$):

$$S_{k_n,k'_n,k_m,k'_m}(t_1,t_2,t_3,t_4) = \left|E_{k_n,k_m}(t_1,t_2) - E_{k_n,k'_m}(t_1,t_3)\right| + \left|E_{k'_n,k_m}(t_4,t_2) - E_{k'_n,k'_m}(t_4,t_3)\right| \leq 2 \quad (28)$$

  With fixed quasispin, e.g. strangeness= +1, the k_q with $q = n, n', m, m'$ are $|K^0\rangle$. The time dependent expectation values $E(t_l, t_r)$ are sufficiently approximated $E^{app}(t_l, t_r) = -cos(\Delta m \Delta t)e^{-\Gamma(t_l+t_r)}$ by neglecting additional terms which vanish due to the smallness of Γ_L and $t_{l,r}$ represent the time. Finally, combining $E^{app}(t_l, t_r)$ and eq.(28) gives Ghirardi, Grassi and Weber's result [23]:

$$\left|e^{-\frac{\Gamma}{2}(t_1+t_2)}cos(\Delta m(t_1-t_2)) - e^{-\frac{\Gamma}{2}(t_1+t_4)}cos(\Delta m(t_1-t_4))\right| + \left|e^{-\frac{\Gamma}{2}(t_2+t_3)}cos(\Delta m(t_2-t_3)) + e^{-\frac{\Gamma}{2}(t_3+t_4)}cos(\Delta m(t_3-t_4))\right| \leq 2 \quad (29)$$

[7] The strangeness eigenstates $|K^0\rangle$ and $|\overline{K^0}\rangle$. The mass (lifetime) eigenstates $|K_S\rangle$ and $|K_L\rangle$. The $\mathcal{CP}$ eigenstates $|K_1^0\rangle$ and $|K_2^0\rangle$

This inequality can be rearranged so that its violation only depends on the ratio: $x = \frac{\Delta m}{\Gamma}$ [12]. In order to violate the inequality, x has to be outside the interval $]0;2[$. However, the experimental value for kaons is right in between with $x_{exper} = 0.95$.

Very Important: [24]The ratio $x = \frac{\Delta m}{\Gamma}$ only applies if the initial state is a $|\psi^-\rangle$. In this case, only the type of meson can be varied. Yet it ($|\psi^-\rangle$) is the most important case, as up to now only $|\psi^-\rangle$ states are available. Regarding the four mesons: K, B, D, B_S only B_S mesons could violate eq. (29) theoretically.

X	MESON SYSTEM
0.95	$K^0\overline{K^0}$
0.77	$B^0\overline{B^0}$
< 0.03	$D^0\overline{D^0}$
> 19	$B_S^0\overline{B_S^0}$

However $|B_S^0\rangle$ and $|\overline{B_S^0}\rangle$ have the quark content $|s\bar{b}\rangle$ and $|\bar{s}b\rangle$, respectively. This renders the active measurement method for kaon quasispin inapplicable for the B-meson system. Passive measurements on the other hand do not make sense for testing Bell inequalities, since decays are left to nature's will and alice and bob cannot decide which decaymode they want to measure.

- Quasispin measurement

  In this case, a test on Wigner like BI eq.(3) seems appropriate, the introduced three bases can provide the three required angles such that inserting in eq.(3) gives:

$$P(|K_S\rangle; |\overline{K^0}\rangle) \leq P(|K_S\rangle; |K_1^0\rangle) + P(|K_1^0\rangle; |\overline{K^0}\rangle) \qquad (30)$$

  This shows violation of Wigner-like BI (which holds under the assumption of locality) is in direct connection with $\mathcal{CP}$-violation. Also, by using the transition amplitudes [12]:

$$\langle\overline{K^0}^{|} K_S\rangle = -\frac{q}{N}, \quad \langle\overline{K^0}|K_1^0\rangle = -\frac{1}{\sqrt{2}}, \quad \langle K_S|K_1^0\rangle = \frac{1}{\sqrt{2}N}(p^* + q^*)$$

  Although a direct measurement of the unphysical state K_1^0 is not possible, a tremendous simplification is possible, which relates the validity of BI with the $\mathcal{CP}$ violation parameter, a testable validity of:

$$|p| \leq |q| \qquad (31)$$

  This again shows the strong connection between $\mathcal{CP}$ violation and entanglement and the importance of testing $\mathcal{CP}$ violation in order to confirm QM. The already mentioned semileptonic decay rule provides such as test for $\mathcal{CP}$ violation.

Let us again consider the semileptonic decays (obeying the $\Delta S = \Delta Q$ rule):
$K^0(d\bar{s}) \rightarrow \pi^-(d\bar{u})l^+\nu_l$
$\overline{K^0}(\bar{d}s) \rightarrow \pi^-(\bar{d}u)l^+\nu_l$

In order to have a comparable quantity for the leptonic charge asymmetry, we will introduce a quantity δ, such that:

$$\delta = \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \overline{\nu_l})}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \overline{\nu_l})} \tag{32}$$

with $l = \mu, e$.
Depending on whether the decay produces a lepton or an antilepton, the K_L decay can be traced back to a K^0 or $\overline{K^0}$ decay.Thus the leptonic charge asymmetry is given by the superpositions given in equation (12) and δ can be expressed by p and q, resulting in:

$$\delta = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \tag{33}$$

using eq. (31) yields:

$$\delta \leq 0 \tag{34}$$

replacing K^0 with $\overline{K^0}$ in eq. (30) gives:

$$|p| \leq |q| \quad , \quad and \quad \delta \geq 0 \tag{35}$$

Finally, combining both gives:

$$|p| = |q| \quad , \quad and \quad \delta = 0 \tag{36}$$

as a criteria equivalent to the Wigner-like BI. However, experimentally the charge asymmetry is nonvanishing:

$$\delta = (3.27 + -0.12)\dot{1}0^{-3} \tag{37}$$

And thus is a clear sign of CP violation, and thereby of BI violation.

3.2.3 Results

Hiesmayr found [24] that concerning a strangeness sensitive measurement out of the four maximally entangeled states:

$$\begin{aligned} |\phi^\pm\rangle &= \frac{1}{2}\left\{|K_S K_S\rangle \pm |K_L K_L\rangle\right\} \\ |\psi^\pm\rangle &= \frac{1}{2}\left\{|K_S K_L\rangle \pm |K_L K_S\rangle\right\} \end{aligned} \tag{38}$$

only $|\phi^-\rangle$ violates the BI slightly with $S = 2.07$. However, by computing S for other states, slightly higher values like $S = 2.1596$ seem achievable.

3.3 Complementarity, Kaons as double slits

Bohr's complementary principle states that one cannot observe an interference pattern and at the same time know about the path taken by the particle. This principle can be expressed by the inequality [22]:

$$\mathcal{P}^2 + \mathcal{V}^2 \leq 1 \tag{39}$$

where $\mathcal{P}$ denotes the predictability $\mathcal{P} = |p_1 - p_2|$ [8], i.e. which path the particle will take and $\mathcal{V}$ denotes the visibility of the interference pattern. Of course $\mathcal{P}$, $\mathcal{V}$ and $p_{1,2}$ can dependent on some external variables. These principles, when applied on the neutral kaon system, have to be slightly changed as through decay time dependence comes into play. The time evolution of $|K^0\rangle$ can be expressed by the two mass eigenstates with [21]:

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}} e^{im_L t - \frac{\Gamma_L}{2} t} \left\{ e^{i\Delta m t + \frac{\Delta\Gamma}{2} t} |K_S\rangle + |K_L\rangle \right\} \tag{40}$$

In case we have a pure $|K^0\rangle$ beam at $t = 0$ eq. (40) shows that both paths, i.e. manifestation as $|K_S\rangle$ or $|K_L\rangle$ seem equally possible and therefore visibility is at its peak. However, since the size of the slits change in time because after a long period finding a $|K_L\rangle$ state is more probable, $\mathcal{V}$ decreases and $\mathcal{P}$ increases. Further analysis [22] showed that eq. (39) contains all the information on the system and is always maximal, i.e. $= 1$ for pure states.

3.4 Decoherence

This section should only constitute a summary to explain why not maximally entangled states can account for maximal violation of BI (except for the case of $|\phi^-\rangle$*.*

Unitary time transformations, which have to be used to describe the kaons propagation from the source of their creation until the point where they are eventually measured or they decay, are derived from the time dependent Schrödinger equation (SE). However, SE only covers closed quantum systems, which the decaying neutral kaon system certainly is not as it interacts with the environment through decay. In order to make sure that a proper time transformations is applicable, one has to include an interaction term with the environment and thereby closing the system again. The master equation includes terms of the system and its interaction with the environment by adding a dissipative term $D[\rho]$:

$$\frac{d}{dt}\rho = -iH\rho + i\rho H^\dagger - D[\rho]$$

with the Hamiltonian H and the density matrix ρ. In order to satisfy the requirement of a closed system, also the decay products have to be included in the state'S descriptions. Naturally, additional terms lead to change in normalization and as time progresses, the weight of surviving states decreases in favour of the weight of the decay products which have no contribution to the violation of BI. Thus finding states which are more resistant concerning dissipation seems key in the quest to violate BI, and also in [24] a possibile state has been presented, which it less entangled, but more resistant to decoherence than the maximally entangeled Bell states. Another figurative explanation for decoherence could go as follows: Decoherence describes the loss of ability to interfere, in other words the fringe visiblity of interference pattern decreases. Taking a case as discussed in section *Complementarity, Kaons as double slits* as time passes and more and more short lived states decay, eventually the long-state-slit will cover most of the

[8] $p_1 + p_2 = 1$ and $p_{1,2}$ represent the probabilities that the particle will take the first or secend path.

figurative double-slit, whereas the short-state-slit will be almost near to extinct. In case the all K_S states vanish, the "‘double slit"’ K^0 will have suffered total decoherence, as the which path-information is maximal since there are only K_L states left. *Regeneration* as discussed before can restore the double slit again.

4 Open Questions

Interesting Questions:

What is the reason for CP-violation? What other manifestation does CP-violation have? Where are the limits to quantum effects? where on the energy scales, where concerning spatial and temporal extension?
Often discussed: CP-violation and entanglement among neutrinos in the early universe. What exactly accounts for the imbalance in matter anti-matter? Neutrino oscillation comparable to strangeness oscillation? if so, can there be an adequate formalism corresponding the neutral kaon system concerning entanglement. What possible applications could be thinked of, which use these effects? Can we find other, more suitably particles to test CP-violation and BI? What about neutrons, as they have much longer lifetimes?

Most important for now:

How to produce states other than $|\psi^-\rangle$ which are highly wished for to test BI, [9] even if it is just a $|\phi^-\rangle$ which violates BI only slightly.

5 Appendix

5.1 Strangeness Number

The introduction of a new quantity called strangeness was supposed to account for a number of surprising events in particle physics as described in section 2.1:

$$S = 2\left(Q - I_3 - \frac{B}{2}\right) \tag{41}$$

where Q denotes the electric charge, which is presumed to be exactly conserved, I_3 is the third component of the isospin and B denotes the baryon number whose conservation rests upon very strong evidence, the proton lifetime [21].

5.2 Time evolution of Kaon states

Time evolution of the $|K_{S,L}\rangle$ states:

$$|K_{S,L}\rangle(t) = e^{-i\lambda_{S,L}t}|K_{S,L}\rangle \tag{42}$$

constructed via the $|K^0\rangle$,$|\overline{K^0}\rangle$ states with:

$$\frac{1}{N}\begin{pmatrix} p & -q \\ p & q \end{pmatrix}\begin{pmatrix} |K^0\rangle \\ |\overline{K^0}\rangle \end{pmatrix} = \begin{pmatrix} |K_S\rangle \\ |K_L\rangle \end{pmatrix} \tag{43}$$

[9] As $|\psi^-\rangle$ suffers from not being able to violate BI with active measurements, as has been pointed out before.

inversion gives:

$$\frac{N^2}{2pq}\begin{pmatrix} q & q \\ -p & p \end{pmatrix}\begin{pmatrix} |K_S\rangle \\ |K_L\rangle \end{pmatrix} = \begin{pmatrix} |K^0\rangle \\ |\overline{K^0}\rangle \end{pmatrix} \tag{44}$$

The time evolution of neutral Kaon states is:

$$\begin{aligned} |K^0\rangle(t) &= e^{-i\lambda_{S,L}t}|K^0\rangle - \frac{q}{p}(1-e^{-i\lambda_{S,L}t})|\overline{K^0}\rangle \\ |\overline{K^0}\rangle(t) &= e^{-i\lambda_{S,L}t}|K^0\rangle - \frac{q}{p}(1-e^{-i\lambda_{S,L}t})|K^0\rangle \end{aligned} \tag{45}$$

With a pure K^0 beam at the beginning ($t = 0$), producable through the strong decay: $\pi^- + p \rightarrow K^0 + \Lambda^0$, the probability of finding a K^0 at time t in the beam is calculated to be [21]:

$$\begin{aligned} P(K^0, t; |K^0|) = |\langle K^0|K^0(t)\rangle|^2 &= \left|\langle K^0|\frac{N}{2p}\left\{e^{-i\lambda_S t}|K_S\rangle + e^{-i\lambda_L t}|K_L\rangle\right\}\rangle\right|^2 \\ &= \left|\frac{1}{2}e^{-i\lambda_S t} - \frac{1}{2}e^{-i\lambda_L t}\right|^2 \\ &= \frac{1}{4}\left\{e^{-i\Gamma_S t} + e^{-i\Gamma_L t} + 2Re\{e^{-i\Delta m t}\}e^{-\Gamma t}\right\} \\ &= \frac{1}{4}\left\{e^{-i\Gamma_S t} + e^{-i\Gamma_L t} + 2cos(\Delta m t)e^{-\Gamma t}\right\} \end{aligned} \tag{46}$$

or finding a $\overline{K^0}$ at time t:

$$\begin{aligned} P(\overline{K^0}, t; |K^0|) = |\langle\overline{K^0}|K^0(t)\rangle|^2 &= \left|\langle\overline{K^0}|\frac{N}{2p}\left\{e^{-i\lambda_S t}|K_S\rangle + e^{-i\lambda_L t}|K_L\rangle\right\}\rangle\right|^2 \\ &= \left|\frac{q}{2p}e^{-i\lambda_S t} - \frac{q}{2p}e^{-i\lambda_L t}\right|^2 \\ &= \frac{1}{4}\frac{|q|^2}{|p|^2}\left\{e^{-i\Gamma_S t} + e^{-i\Gamma_L t} - 2cos(\Delta m t)e^{-\Gamma t}\right\} \end{aligned} \tag{47}$$

with: $\Delta m = m_L - m_S$ and $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)$

5.3 Regeneration process

Under negligence of $\mathcal{CP}$ violation and after a reasonable amount of time, an initially pure K^0 beam evolved into a almost pure K_L beam. The piece of matter then "‘acts"’ on the K_L states:

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K^0}\rangle) \rightarrow \frac{1}{\sqrt{2}}(a|K^0\rangle + b|\overline{K^0}\rangle) \tag{48}$$

with $b < a < 1$. Reformulating the state gives:

$$\frac{1}{\sqrt{2}}(a|K^0\rangle + b|\overline{K^0}\rangle) = \frac{1}{\sqrt{2}}(\frac{c}{2}|K^0\rangle + \frac{-c}{2}|\overline{K^0}\rangle + \frac{d}{2}|K^0\rangle + \frac{d}{2}|\overline{K^0}\rangle)$$

with $c = a - b$ and $d = a + b$ and rearranging:

$$... = \frac{1}{2\sqrt{2}}(c|K^0\rangle - c|\overline{K^0}\rangle) + \frac{1}{2\sqrt{2}}(d|K^0\rangle + d|\overline{K^0}\rangle)$$

yields:

$$... = \frac{c}{2}|K_S\rangle + \frac{d}{2}|K_L\rangle \tag{49}$$

of course c and d depend on several specific material parameters. Eq. (49) shows that $|K_S\rangle$ states are regenerated. The more effectively the piece of matter "‘absorbs"’ the $|\overline{K^0}\rangle$ states, the higher the rate of regeneration.

5.4 Deduction of Bell inequality for spin$\frac{1}{2}$ fermions

Following Gedankenexperiment should help to understand the Bell inequality: A system of two spin $\frac{1}{2}$ particles is prepared in a way, that both particles fly off in different diraction towards one measuring instrument on each side, Alice and Bob, measuring the spin along two individual components $\hat{a}$ and $\hat{b}$, respectively. LHV suggests that a probability density $\rho(\lambda)$, with hidden variables λ and normalization such that:

$$\int d\lambda \rho(\lambda) = 1 \tag{50}$$

can account for all the predictions of quantum mechanics. In the course of the deduction, there shall be two different settings at Alice denoted by $a_{1,2}$ and likewise for Bob's settings, and $\overline{A}$ and $\overline{B}$ shall represent the mean values at A and B, respectively. The results of a measurement performed by Alice or Bob can have positive or negative results, i.e. 1 or -1. Let us alo keep in mind, that as this deduction refers to a LHV, the outcome of measurements performed at A are only dependent of the settings at Alice, i.e. especially measurements performed at B do not affect measurements performed at A and vice versa. The expectation of such a "‘double measurement"’ is given by:

$$E(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) \tag{51}$$

$$E(\mathbf{a}, \mathbf{b}) = \langle \psi | \sigma \cdot \mathbf{a} \otimes \sigma \cdot \mathbf{b} | \psi \rangle = -\mathbf{a} \cdot \mathbf{b} \tag{52}$$

The following steps will not be accompanied by further comment, as it is described in more detail [7].

$$\begin{aligned} E(a_1, b_1) - E(a_1, b_2) = & \int d\lambda \rho(\lambda) A(a_1, \lambda) B(b_1, \lambda) \{1 \pm A(a_2, \lambda) B(b_2, \lambda)\} \\ & - \int d\lambda \rho(\lambda) A(a_1, \lambda) B(b_2, \lambda) \{1 \pm A(a_2, \lambda) B(b_1, \lambda)\} \end{aligned} \tag{53}$$

$$\begin{aligned} |E(a_1, b_1) - E(a_1, b_2)| \leq & \int d\lambda \rho(\lambda) \{1 \pm A(a_2, \lambda) B(b_2, \lambda)\} \\ & + \int d\lambda \rho(\lambda) \{1 \pm A(a_2, \lambda) B(b_1, \lambda)\} \end{aligned} \tag{54}$$

$$|E(a_1, b_1) - E(a_1, b_2)| \leq 2 \pm |E(a_2, b_2) - E(a_2, b_1)| \tag{55}$$

final rearranging leads to the CHSH-inequality:

$$S = |E(a_1, b_1) - E(a_1, b_2)| + |E(a_2, b_2) - E(a_2, b_1)| \leq 2 \tag{56}$$

5.5 Entanglement witness

An Entanglement Witnesses (EW) is applied to ascertain whether a given state is entangled or not. EW are represented by hermitian operators $\mathcal{W}$, i.e. they are observables and have real eigenvalues. If ρ denotes the probability density matrix and $\mathcal{S}$ the set of seperable states, following statements apply to EW [15]:

$$\forall \rho \in \mathcal{S}, \quad tr(\mathcal{W}\rho) \geq 0 \tag{57}$$

At least one entangled state satisfies:

$$\exists \rho \notin \mathcal{S}, \quad tr(\mathcal{W}\rho) < 0 \tag{58}$$

Each entangled state has at least one EW to detect it with:

$$\forall \rho \notin \mathcal{S}, \quad \exists \mathcal{W} \quad with \quad tr(\mathcal{W}\rho) < 0 \tag{59}$$

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