Evolutionary dynamics of two linked bodies' plane motion

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1. Introduction

The aim of this work is to apply one of the little-known phenomena of the solid dynamics – their evolutionary behavior – due to the forces of internal dissipation to the description of the Earth-Moon system. In terrestrial systems these forces appear under free or induced vibrations of deformable bodies, accompanied by dissipation of energy due to mechanical hysteresis. The power of such dispersion is comparable to the power of external friction forces, so the process of dissipation is "fast" (duration of damping of free oscillations is estimated in tens periods of oscillations). A different mechanism of internal dissipation takes place in outer space, where there is no external friction, and activators of hysteresis phenomena in the body are the forces of inertia when it is non-uniform rotation, and also the external gravitational field, if present.

2. Main body

The problem of the effect on the energy dissipation due to the volume forces and the forces of inertia fields' changes over time is considered on the example of a mechanical system, presented in Fig. 1. The system consists of two homogeneous discs; their centers are connected with non-deformable massless rod l. It is assumed that radius r_m and mass m of one disc are much smaller than radius r_M and mass M of the other.

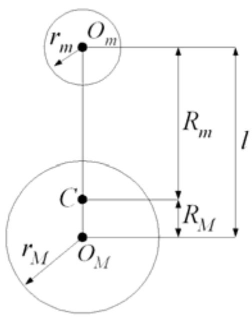

Fig. 1

There are no external forces and moments, so the system's center of mass, denoted by C, is stationary (or moving with velocity of constant magnitude and direction). Disc r_M creates a gravitational field of intensity $\underline{g}$. It is also assumed that the distance l between the centers of discs is much larger than r_m, so the gravitational field near the disc r_m is considered to be homogeneous and directed along the rod l.

The main hypothesis when solving the problem is to abandon the model of a rigid body. It is assumed that the rigid shell of the disc is filled with a dissipative medium, which is able to perform micromotions under the influence of non-stationary field of volume and inertia forces. The scale of these movements is much smaller than the scale of body's motion, so when deriving the equations of motion elementary volumes' micromotions can be neglected. Their influence on evolution of the system's movement is taken into account integrally.

To determine the loss of energy, a dissipative function S is constructed [2], which is similar to the Rayleigh function

$$S = \frac{1}{2}\int_V b\left|\dot{\underline{w}}\right|^2 dV \ , \tag{1}$$

where b is dissipative coefficient, $\underline{w} = -\underline{\widetilde{w}} + \underline{g}$, $\underline{\widetilde{w}}$ is absolute acceleration of a volume element (the minus sign corresponds to the force of inertia), $\underline{g}$ is density of the volume force. To simplify the calculations, let us take into account the energy loss in the disc r_m only.

Let us introduce a coordinate system with axes $O_m\xi$ and $O_m\eta$, rigidly connected with the disc r_m (Fig.2). In the beginning $\xi = -y + \left|CO_m\right|,\ \ \eta = x$, the rod is directed along the axis Cy. Here Cx and Cy are the axis of the fixed coordinate system with the origin at the center of mass C. Angle φ characterizes the rotation of the disc r_m relative to the rod, angle θ defines the rotation of the rod (and the entire system) relative to the initial position.

Projections of the total acceleration $\underline{w}$ of elementary volume to the axis $O_m\xi$ and $O_m\eta$

$$\begin{cases} w_\xi = \rho(\dot{\varphi}+\dot{\theta})^2\cos\alpha + \rho(\ddot{\varphi}+\ddot{\theta})\sin\alpha - \dot{\theta}^2 R_m\cos\varphi + \ddot{\theta}R_m\sin\varphi + g\cos\varphi \\ w_\eta = \rho(\dot{\varphi}+\dot{\theta})^2\sin\alpha - \rho(\ddot{\varphi}+\ddot{\theta})\cos\alpha + \dot{\theta}^2 R_m\sin\varphi + \ddot{\theta}R_m\cos\varphi - g\sin\varphi \end{cases},$$

where ρ and α are polar coordinates of a volume element (Fig.2).

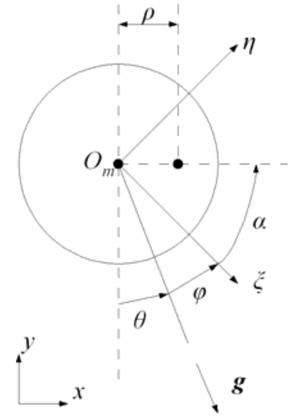

Fig. 2

Thus $S = bs(\dot{\varphi},\ddot{\varphi},\dddot{\varphi},\dot{\theta},\ddot{\theta},\dddot{\theta},g)$, so the dissipative function depends on the third derivatives of generalized coordinates, as well as the intensity of the gravitational field.

Let us derive the equations of motion for the considered system [1]. Kinetic energy is

$$T = \frac{1}{2}J_m\left(\dot{\varphi}+\dot{\theta}\right)^2 + \frac{1}{2}mR_m{}^2\dot{\theta}^2 + \frac{1}{2}J_M\dot{\theta}^2 + \frac{1}{2}MR_M{}^2\dot{\theta}^2, \qquad (5)$$

where J_m and J_M are discs' moments of inertia, and for the angular momentum, calculated relative to the axis, perpendicular to the plane of the drawing and passing through point C, we have

$$L = J_m\dot{\varphi} + \left(J_m + mR_m{}^2 + J_M + MR_M{}^2\right)\dot{\theta}. \qquad (6)$$

Assuming the angular momentum L of the system known and that initially the smaller disc rotates around its center O_m with angular velocity $\omega_0 = L/J_m$, we get from (6): $L = J_m\omega_0 \Rightarrow \dot{\theta} = -\varepsilon(\dot{\varphi}-\omega_0)$, where $\varepsilon = J_m \Big/ \left(J_m + mR_m{}^2 + J_M + MR_M{}^2\right)$ is a small parameter, as radius r_m and mass m of one disc are much smaller then radius r_M and mass M of the other. Thus, we obtain the system with one degree of freedom, so, in terms of the problem, Lagrange equation of the second kind is according to [2]

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}} - \frac{\partial T}{\partial \varphi} = -\frac{\partial S}{\partial \dot{\varphi}} \qquad (7)$$

The potential energy is not included in the equation (7), because it is constant (the distance between the discs is constant and there are no external fields). Given (4) and (5), the differential equation of motion takes the form

$$\tilde{J}\ddot{\varphi} = -bs(\dot{\varphi},\ddot{\varphi},\dddot{\varphi},\varepsilon,g), \qquad (8)$$

where $\tilde{J} = J_m(1-\varepsilon)^2 + (J_M + mR_m{}^2 + MR_M{}^2)\varepsilon$. Equation (8), firstly, contains a third time derivative of the angle φ, and, secondly, in the absence of dissipation, it degenerates into $\ddot{\varphi} = 0 \Rightarrow \dot{\varphi} = \omega_0,\ \dot{\theta} = 0$. Thus, in the absence of internal dissipation, redistribution of angular momentum does not occur in the system.

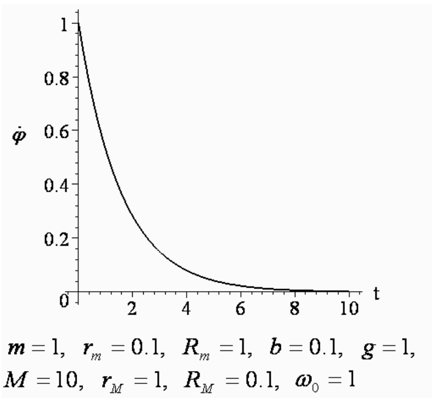

Fig. 3

If $b \neq 0$, then in the limit state $\dot{\varphi} = 0,\ \dot{\theta} = \varepsilon\omega_0$, and the length of the transient process is inversely proportional to b.

The results of the numerical solutions and the values of system parameters, corresponding to this solution are shown in Fig. 3.

The calculations show that the magnitude of the energy loss does not depend on the dissipative coefficient (if it is not equal to zero).

Separate calculations performed for the case $g = 0$ have shown, that in the absence of volume force for any value of $b \neq 0$, redistribution of angular momentum does not occur in the system. It is non-stationary gravitational field, which is internal to the system, but external to the dissipative disc, causes the evolutionary motion.

To model the Earth-Moon system, a similar problem, but without a massless rod, which will allow the discs, representing the Earth and the Moon to get either away from each other or close to each other, is to be considered.

4. References

[1] A.I. Lurie, *Analytical Mechanics*, Fizmatlit, Moscow (1961).
[2] B.A. Smolnikov, "Evolutionary dynamics of pendulum systems", *Theory of Mechanisms and Machines*, №1, Vol. 6, P. 41-47 (2008).